AP Calculus AB
**Review 07**, No Calculator

Complete all the following on notebook paper.

_____ 1.

$$\frac{d}{dx}\left(\ln e^{2x}\right) =$$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) $1$ (E) $2$

_____ 2.

The area of the region bounded by the curve $y = e^{2x}$, the $x$-axis, the $y$-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

_____ 3.

If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of $x$ ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

_____ 4.

$\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) $-1$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $1$

(E) none of the above

_____ 5.

A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the $x$-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of $m$ .

(B) increases as $m$ increases.

(C) decreases as $m$ increases.

(D) decreases as $m$ increases when $m < \frac{1}{2}$; increases as $m$ increases when $m > \frac{1}{2}$.

(E) increases as $m$ increases when $m < \frac{1}{2}$; decreases as $m$ increases when $m > \frac{1}{2}$.

_____ 6.

If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

_____ 7.

If $y = x^2 e^x$ , then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x\left(x + 2e^x\right)$ (C) $xe^x\left(x + 2\right)$

(D) $2x + e^x$ (E) $2x + e$

_____ 8.

$\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

_____ 9.

If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) $e^{-x-1}$ (C) $e^{1-x}$ (D) $e^{-x}$ (E) $-e^x$

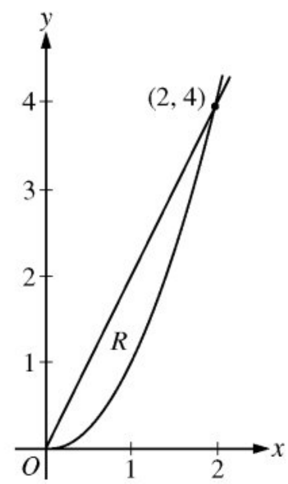
_____ 10.

If a function $f$ is continuous for all $x$ and if $f$ has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(A) The graph of $f$ has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f'(-1) = 0$
(C) The graph of $f$ has a horizontal asymptote.
(D) The graph of $f$ has a horizontal tangent line at $x = 3$.
(E) The graph of $f$ intersects both axes.

11. 2009—AB4

Let $R$ be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.




(a) Find the area of $R$.
(b) The region $R$ is the base of a solid. For this solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.
(c) Another solid has the same base $R$. For this solid, the cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

12. 2009—AB5

| $x$ | 2 | 3 | 5 | 8 | 13 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | –2 | 3 | 6 |

Let $f$ be a function that is twice differentiable for all real numbers. The table above gives values of $f$ for selected points in the closed interval $2 \le x \le 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all $x$ in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of $f$ at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of $f$ on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.