Complete all the following on notebook paper.

1. \[ \frac{d}{dx} \left( \ln e^{2x} \right) = \]
   (A) \( \frac{1}{e^{2x}} \) \quad (B) \( \frac{2}{e^{2x}} \) \quad (C) \( 2x \) \quad (D) \( 1 \) \quad (E) \( 2 \)

2. The area of the region bounded by the curve \( y = e^{2x} \), the \( x \)-axis, the \( y \)-axis, and the line \( x = 2 \) is equal to
   (A) \( \frac{e^4}{2} - e \) \quad (B) \( \frac{e^4}{2} - 1 \) \quad (C) \( \frac{e^4}{2} - \frac{1}{2} \)
   (D) \( 2e^4 - e \) \quad (E) \( 2e^4 - 2 \)

3. If \( \sin x = e^y \), \( 0 < x < \pi \), what is \( \frac{dy}{dx} \) in terms of \( x \) ?
   (A) \( -\tan x \) \quad (B) \( -\cot x \) \quad (C) \( \cot x \) \quad (D) \( \tan x \) \quad (E) \( \csc x \)

4. \[ \int_{0}^{1} \sqrt{x^2 - 2x + 1} \, dx \]
   (A) \( -1 \)
   (B) \( -\frac{1}{2} \)
   (C) \( \frac{1}{2} \)
   (D) \( 1 \)
   (E) none of the above
5. A region in the plane is bounded by the graph of \( y = \frac{1}{x} \), the x-axis, the line \( x = m \), and the line \( x = 2m \), \( m > 0 \). The area of this region

(A) is independent of \( m \).
(B) increases as \( m \) increases.
(C) decreases as \( m \) increases.
(D) decreases as \( m \) increases when \( m < \frac{1}{2} \); increases as \( m \) increases when \( m > \frac{1}{2} \).
(E) increases as \( m \) increases when \( m < \frac{1}{2} \); decreases as \( m \) increases when \( m > \frac{1}{2} \).

6. If \( \frac{dy}{dx} = \tan x \), then \( y = \)

(A) \( \frac{1}{2} \tan^2 x + C \)
(B) \( \sec^2 x + C \)
(C) \( \ln | \sec x | + C \)
(D) \( \ln | \cos x | + C \)
(E) \( \sec x \tan x + C \)

7. If \( y = x^2 e^x \), then \( \frac{dy}{dx} = \)

(A) \( 2xe^x \)
(B) \( x(2 + 2e^x) \)
(C) \( xe^x(x + 2) \)
(D) \( 2x + e^x \)
(E) \( 2x + e \)

8. \( \int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} \, dx = \)

(A) \( \ln \sqrt{2} \)
(B) \( \ln \frac{\pi}{4} \)
(C) \( \ln \sqrt{3} \)
(D) \( \ln \frac{\sqrt{3}}{2} \)
(E) \( \ln e \)

9. If \( f''(x) = -f(x) \) and \( f(1) = 1 \), then \( f(x) = \)

(A) \( \frac{1}{2} e^{-2x+2} \)
(B) \( e^{-x-1} \)
(C) \( e^{1-x} \)
(D) \( e^{-x} \)
(E) \( -e^x \)
10. If a function $f$ is continuous for all $x$ and if $f$ has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true?

(A) The graph of $f$ has a point of inflection somewhere between $x = -1$ and $x = 3$.
(B) $f''(-1) = 0$
(C) The graph of $f$ has a horizontal asymptote.
(D) The graph of $f$ has a horizontal tangent line at $x = 3$.
(E) The graph of $f$ intersects both axes.

11. 2009—AB4

Let $R$ be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

(a) Find the area of $R$.
(b) The region $R$ is the base of a solid. For this solid, at each $x$ the cross section perpendicular to the $x$-axis has area $A(x) = \sin\left(\frac{\pi}{2} x\right)$. Find the volume of the solid.
(c) Another solid has the same base $R$. For this solid, the cross sections perpendicular to the $y$-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

12. 2009—AB5

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>4</td>
<td>-2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Let $f$ be a function that is twice differentiable for all real numbers. The table above gives values of $f$ for selected points in the closed interval $2 \leq x \leq 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_{2}^{13} (3 - 5f'(x)) \, dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_{2}^{13} f(x) \, dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all $x$ in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of $f$ at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of $f$ on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$. 