

Complete all the following on notebook paper.

_____ 1.

$$\frac{1}{2} \int e^{\frac{t}{2}} dt =$$

- (A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^t + C$

_____ 2.

If a, b, c, d , and e are real numbers and $a \neq 0$, then the polynomial equation $ax^7 + bx^5 + cx^3 + dx + e = 0$ has

- (A) only one real root.
(B) at least one real root.
(C) an odd number of nonreal roots.
(D) no real roots.
(E) no positive real roots.

_____ 3.

What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

- (A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

_____ 4.

Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

- (A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$

_____ 5.

At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

- (A) 32 (B) 48 (C) 64 (D) 96 (E) 192

_____ 6.

The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

- (A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

_____ 7.

$$\int \frac{x^2}{e^{x^3}} dx =$$

- (A) $-\frac{1}{3} \ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$
(D) $\frac{1}{3} \ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

_____ 8.

If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

- (A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

_____ 9.

If n is a non-negative integer, then $\int_0^1 x^n dx = \int_0^1 (1-x)^n dx$ for

- (A) no n (B) n even, only (C) n odd, only
(D) nonzero n , only (E) all n

_____ 10.

If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \leq x \leq 2, \\ f(x) = x^2 & \text{elsewhere,} \end{cases}$ then $\int_{-1}^3 f(x) dx$ is a number between

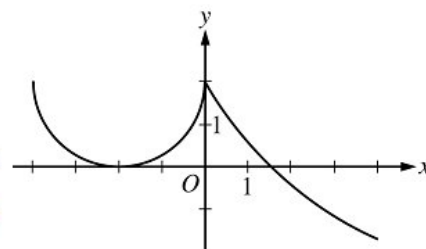
- (A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

11. 2009—AB6

The derivative of a function f is defined by

$$f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$$

The graph of the continuous function f' , shown in the figure above, has x -intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \leq x \leq 0$ is a semicircle, and $f(0) = 5$.

Graph of f'

- For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.
- Find $f(-4)$ and $f(4)$.
- For $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum. Justify your answer.

12. 2009B—AB4

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.

- Find the area of R .
- The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are squares. Find the volume of this solid.
- Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

