AP Calculus AB
Review 09, No Calculator

Complete all the following on notebook paper.

_____ 1.

What are all values of k for which the graph of $y = x^3 - 3x^2 + k$ will have three distinct x-intercepts?

(A) All $k > 0$ (B) All $k < 4$ (C) $k = 0, 4$ (D) $0 < k < 4$ (E) All k

_____ 2.

$\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3)+C$ (B) $\cos(2x+3)+C$ (C) $-\cos(2x+3)+C$

(D) $-\frac{1}{2}\cos(2x+3)+C$ (E) $-\frac{1}{5}\cos(2x+3)+C$

_____ 3.

If $\frac{d}{dx}(f(x)) = g(x)$ and $\frac{d}{dx}(g(x)) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

(A) $f\left(x^6\right)$ (B) $g\left(x^3\right)$ (C) $3x^2 g\left(x^3\right)$

(D) $9x^4 f\left(x^6\right) + 6x\, g\left(x^3\right)$ (E) $f\left(x^6\right) + g\left(x^3\right)$

_____ 4.

$\int \left(x^3 - 3x\right) dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

_____ 5.

If $f(x)=x^3+3x^2+4x+5$ and $g(x)=5$, then $g\left(f(x)\right)=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 1125

(D) 225 (E) 5

_____ 6.

The slope of the line tangent to the graph of $y=\ln\left(x^2\right)$ at $x=e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

_____ 7.

If $f(x)=x+\sin x$, then $f'(x)=$

(A) $1+\cos x$ (B) $1-\cos x$ (C) $\cos x$

(D) $\sin x-x\cos x$ (E) $\sin x+x\cos x$

_____ 8.

If $f(x)=e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y=0$ (B) $x=0$ (C) $y=x$ (D) $y=-x$ (E) $y=1$

_____ 9.

If $f(x)=\frac{x-1}{x+1}$ for all $x\neq -1$, then $f'(1)=$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

_____ 10.

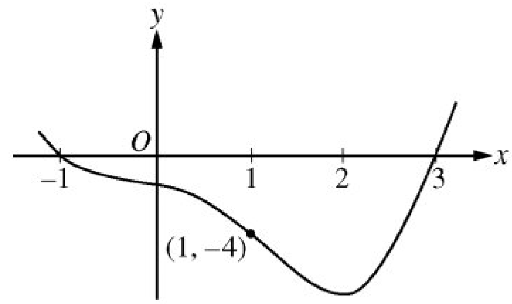
Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y=\frac{x+1}{x}$ (B) $y=-x^5+3x$ (C) $y=x^4-2x^2+6$

(D) $y=(x-1)^3+1$ (E) $y=\left(x^2+1\right)^2-1$

11. 2009B—AB5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

12. 2009B—AB6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.