

AP Calculus AB
Review 11, No Calculator

Complete all the following on notebook paper.

_____ 1.
 $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$

(B) $\frac{-2}{\sqrt{4x^2-1}}$

(C) $\frac{1}{2\sqrt{1-4x^2}}$

(D) $\frac{2}{\sqrt{1-4x^2}}$

(E) $\frac{2}{\sqrt{4x^2-1}}$

_____ 2.

Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function?

- (A) The function f is periodic.
- (B) The graph of f is symmetric with respect to the y -axis.
- (C) The graph of f is concave up.
- (D) The function f is a strictly increasing function.
- (E) The function f is continuous.

_____ 3.

If F and f are continuous functions such that $F'(x) = f(x)$ for all x , then $\int_a^b f(x) dx$ is

- (A) $F'(a) - F'(b)$
- (B) $F'(b) - F'(a)$
- (C) $F(a) - F(b)$
- (D) $F(b) - F(a)$
- (E) none of the above

_____ 4.

$$\int_0^1 (x+1)e^{x^2+2x} dx =$$

- (A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

_____ 5.

Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

- (A) $x > 0$
- (B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
- (C) $-2 < x < 0$ or $x > 2$
- (D) $x > \sqrt{2}$
- (E) $-2 < x < 2$

_____ 6.

$\lim_{h \rightarrow 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is

- (A) e^2
- (B) 1
- (C) $\frac{1}{2}$
- (D) 0
- (E) nonexistent

_____ 7.

Let $f(x) = \cos(\arctan x)$. What is the range of f ?

- (A) $\left\{x \mid -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$
- (B) $\{x \mid 0 < x \leq 1\}$
- (C) $\{x \mid 0 \leq x \leq 1\}$
- (D) $\{x \mid -1 < x < 1\}$
- (E) $\{x \mid -1 \leq x \leq 1\}$

_____ 8.

$\int_0^{\pi/4} \tan^2 x \, dx =$

- (A) $\frac{\pi}{4} - 1$
- (B) $1 - \frac{\pi}{4}$
- (C) $\frac{1}{3}$
- (D) $\sqrt{2} - 1$
- (E) $\frac{\pi}{4} + 1$

_____ 9.

The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V ? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

- (A) 10π
- (B) 12π
- (C) 22.5π
- (D) 25π
- (E) 30π

_____ 10.

$$\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}} dx =$$

- (A) $1 - \frac{\sqrt{3}}{2}$ (B) $\frac{1}{2} \ln \frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6} - 1$ (E) $2 - \sqrt{3}$

11. 2008-AB6

Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by

$$f'(x) = \frac{1 - \ln x}{x^2}.$$

- (a) Write an equation for the line tangent to the graph of f at $x = e^2$.
(b) Find the x -coordinate of the critical point of f . Determine whether this point is a relative minimum, a relative maximum, or neither for the function f . Justify your answer.
(c) The graph of the function f has exactly one point of inflection. Find the x -coordinate of this point.
(d) Find $\lim_{x \rightarrow 0^+} f(x)$.

12. 2008-AB4B

The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2} dt$ and $g(x) = f(\sin x)$.

- (a) Find $f'(x)$ and $g'(x)$.
(b) Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.
(c) Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \leq x \leq \pi$. Justify your answer.