AP Calculus AB
Review 12, No Calculator

Complete all the following on notebook paper.

_____ 1.

A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t-3t^2$. What is the *total* distance covered by the point between $t=1$ and $t=2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

_____ 2.

Let $f(x)=\left|\sin x-\frac{1}{2}\right|$. The maximum value attained by f is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$

_____ 3.

$\int_1^2 \frac{x-4}{x^2}dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

_____ 4.

If $\log_a\left(2^a\right)=\frac{a}{4}$, then $a=$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

_____ 5.

$\int \frac{5}{1+x^2}dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$ (E) $5\ln\left(1+x^2\right)+C$

_____ 6.

Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$

(B) $-f'(x_0)$

(C) $\dfrac{1}{f'(x_0)}$

(D) $\dfrac{-1}{f'(x_0)}$

(E) None of the above

_____ 7.

The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

_____ 8.

The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \dfrac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\dfrac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\dfrac{8\pi}{3}$

_____ 9.

If $y = e^{nx}$, then $\dfrac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) ne^{nx} (D) $n^n e^x$ (E) $n!e^x$

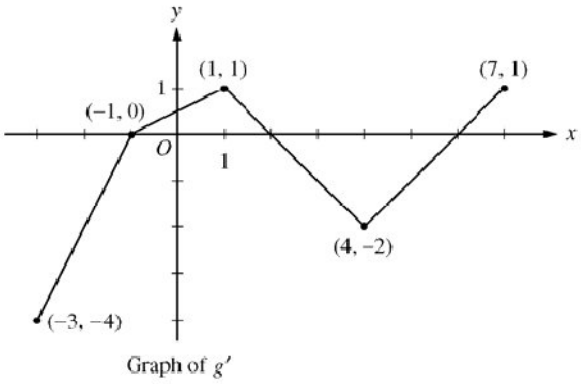
_____ 10.

If $\dfrac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

11. 2008-AB5B

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

12. 2008-AB6B

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2\left(y^3+1\right)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.