AP Calculus AB
Review 14, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.

$\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

_____ 2.

If $f(x) = x$, then $f'(5) =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$

_____ 3.

If $y = 10^{\left(x^2-1\right)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{\left(x^2-1\right)}$ (B) $(2x)10^{\left(x^2-1\right)}$ (C) $\left(x^2-1\right)10^{\left(x^2-2\right)}$

(D) $2x(\ln 10)10^{\left(x^2-1\right)}$ (E) $x^2(\ln 10)10^{\left(x^2-1\right)}$

_____ 4.

The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

_____ 5.

If $f(g(x)) = \ln\left(x^2+4\right)$, $f(x) = \ln\left(x^2\right)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

_____6.

If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

_____ 7.

The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

_____ 8.

The domain of the function defined by $f(x) = \ln\left(x^2 - 4\right)$ is the set of all real numbers x such that

(A) $|x| < 2$ (B) $|x| \le 2$ (C) $|x| > 2$ (D) $|x| \ge 2$ (E) x is a real number

_____ 9.

The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

_____ 10.

If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

Free Response CALCULATOR PERMITTED

11. 2005-AB2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

12. 2005-AB3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.