AP Calculus AB
Review 17, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.

If $\lim_{x\to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$

(E) None of the above

_____ 2.

$$\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$$

(A) $\dfrac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\dfrac{x}{\sqrt{1+x^2}}-\dfrac{1}{\sqrt{5}}$ (E) $\dfrac{1}{2\sqrt{1+x^2}}-\dfrac{1}{2\sqrt{5}}$

_____ 3.

An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

_____ 4.

The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0,2]$ is

(A) $\dfrac{26}{9}$ (B) $\dfrac{13}{3}$ (C) $\dfrac{26}{3}$ (D) 13 (E) 26

_____ 5.

The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

_____ 6.

The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9

_____ 7.

At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

(A) -1 only (B) 0 only (C) 1 only (D) -1 and 1 only (E) -1, 0 and 1

_____ 8.

If $f(x) = \frac{x}{\tan x}$, then $f'\left(\frac{\pi}{4}\right) =$

(A) 2 (B) $\frac{1}{2}$ (C) $1 + \frac{\pi}{2}$ (D) $\frac{\pi}{2} - 1$ (E) $1 - \frac{\pi}{2}$

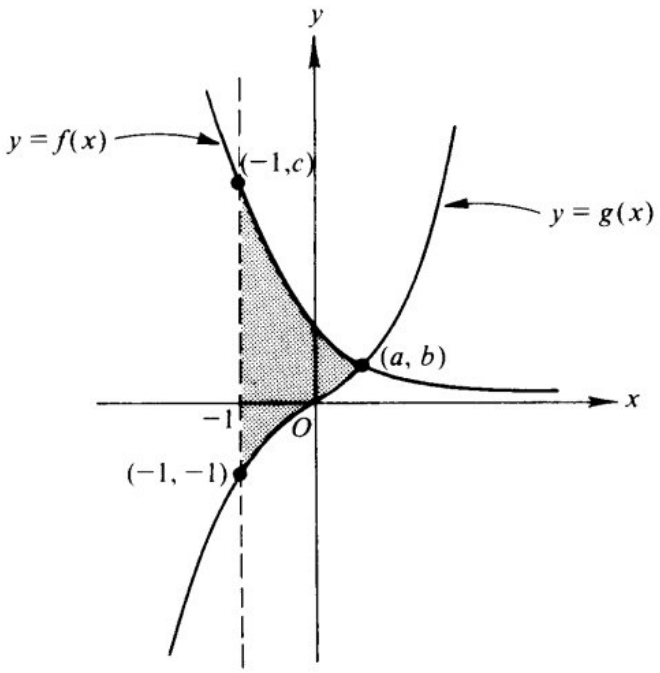
_____ 9.

Which of the following is equal to $\int \frac{1}{\sqrt{25 - x^2}}\,dx$?

(A) $\arcsin\frac{x}{5} + C$ (B) $\arcsin x + C$ (C) $\frac{1}{5}\arcsin\frac{x}{5} + C$

(D) $\sqrt{25 - x^2} + C$ (E) $2\sqrt{25 - x^2} + C$

_____ 10.



The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a,b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

(A) $\int_0^a \big(f(x) - g(x)\big)dx + \int_{-1}^0 \big(f(x) + g(x)\big)dx$

(B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_{-1}^c \big(f(x) - g(x)\big)dx$

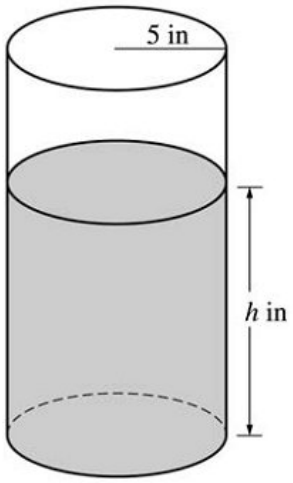
(D) $\int_{-1}^a \big(f(x) - g(x)\big)dx$

(E) $\int_{-1}^a \big(|f(x)| - |g(x)|\big)dx$

II. Free Response

11. 2003-AB5 (No Calculator)

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)



(a) Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.

(b) Given that $h = 17$ at time $t = 0$, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ for h as a function of t.

(c) At what time t is the coffeepot empty?

12. 2003-AB6 (No Calculator)

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?