

Complete all the following on notebook paper.

_____ 1.

Let f be a continuous function on the closed interval $[0, 2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\int_0^2 f(x) dx$ is

- (A) 0 (B) 2 (C) 4 (D) 8 (E) 16

_____ 2.

$$\int_0^1 (3x-2)^2 dx =$$

- (A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

_____ 3.

If $y = 2 \cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2} =$

- (A) $-8 \cos\left(\frac{x}{2}\right)$ (B) $-2 \cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2} \cos\left(\frac{x}{2}\right)$

_____ 4.

$$\int_2^3 \frac{x}{x^2+1} dx =$$

- (A) $\frac{1}{2} \ln \frac{3}{2}$ (B) $\frac{1}{2} \ln 2$ (C) $\ln 2$ (D) $2 \ln 2$ (E) $\frac{1}{2} \ln 5$

_____ 5.

The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

- (A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

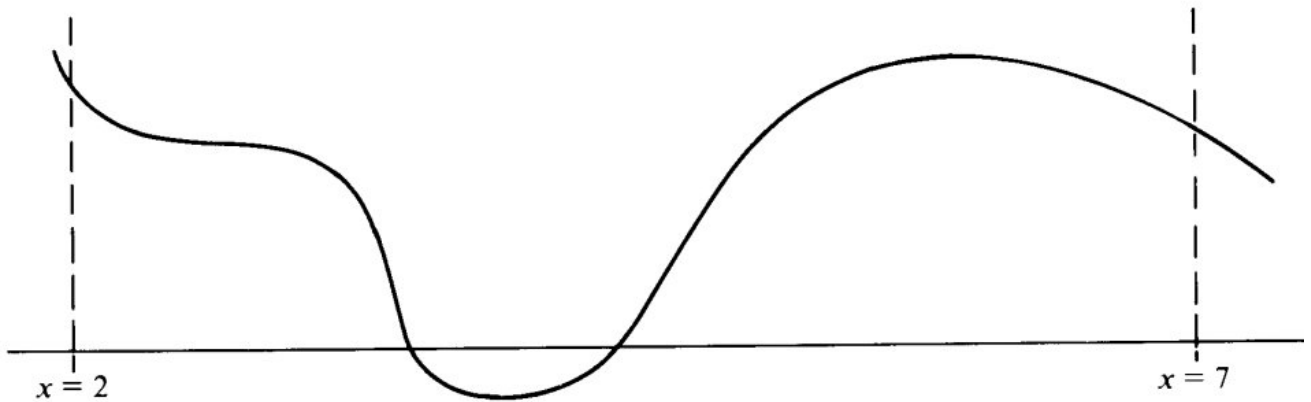
6.

Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a) = f(b) = 1$, which of the following must be true for at least one value of x between a and b ?

- I. $f(x) = 0$
- II. $f'(x) = 0$
- III. $f''(x) = 0$

- (A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

7.



The graph of $y = f(x)$ on the closed interval $[2, 7]$ is shown above. How many points of inflection does this graph have on this interval?

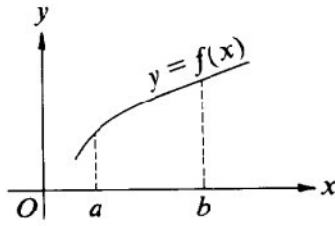
- (A) One (B) Two (C) Three (D) Four (E) Five

8.

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} \text{ is}$$

- (A) 0 (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\pi}{4}$ (D) 1 (E) nonexistent

9.



If f is the continuous, strictly increasing function on the interval $a \leq x \leq b$ as shown above, which of the following must be true?

I. $\int_a^b f(x) dx < f(b)(b-a)$

II. $\int_a^b f(x) dx > f(a)(b-a)$

III. $\int_a^b f(x) dx = f(c)(b-a)$ for some number c such that $a < c < b$

- (A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

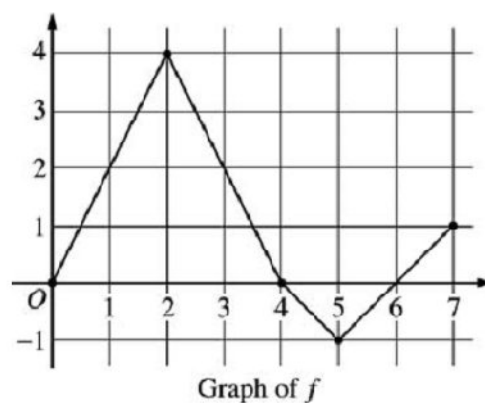
10.

An equation of the line normal to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

- (A) $4x + y = -10$ (B) $x - 4y = 23$ (C) $4x - y = 2$ (D) $x + 4y = 25$ (E) $x + 4y = -25$

11. 2003—AB5B (No Calculator)

Let f be a function defined on the closed interval $[0, 7]$. The graph of f , consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t) dt$.



- Find $g(3)$, $g'(3)$, and $g''(3)$.
- Find the average rate of change of g on the interval $0 \leq x \leq 3$.
- For how many values c , where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.
- Find the x -coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

12. 2003-AB6B (No Calculator)

Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x , where $f(3) = 25$.

- Find $f''(3)$.
- Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25$.