AP Calculus AB
Review 20, No Calculator Permitted on MC

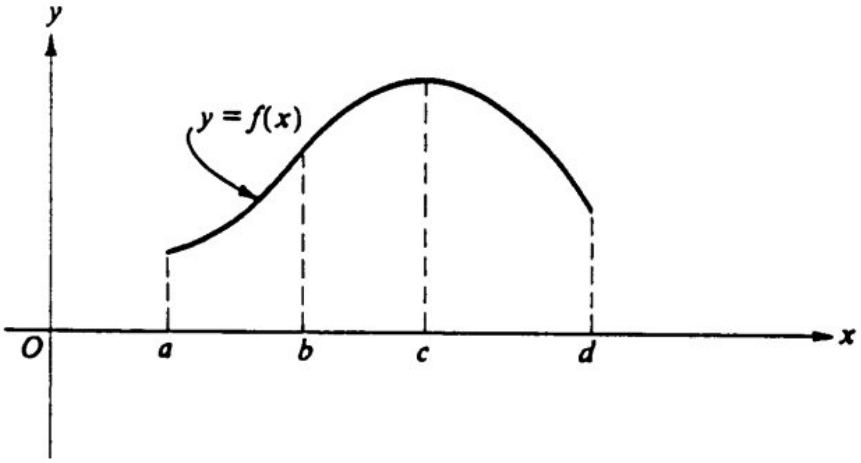
Complete all the following on notebook paper.

_____ 1.

If $y = \dfrac{\ln x}{x}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x^2}$ (C) $\dfrac{\ln x - 1}{x^2}$ (D) $\dfrac{1 - \ln x}{x^2}$ (E) $\dfrac{1 + \ln x}{x^2}$

_____ 2.



The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I, II (B) I, III (C) I only (D) II only (E) III only

_____ 3.

If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\dfrac{dy}{dx}$ is

(A) $\dfrac{3}{2}$ (B) $\dfrac{1}{2}$ (C) 0 (D) $-\dfrac{3}{2}$ (E) nonexistent

_____ 4.

If $\int_0^k \left(2kx - x^2\right)dx = 18$, then $k =$

(A) –9 (B) –3 (C) 3 (D) 9 (E) 18

_____ 5.

An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x + 1$

(D) $y = 2x - 3$ (E) $y = 7x - 8$

_____ 6.

If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

_____ 7.

If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c) - f(0)$ (B) $|f(c) - f(0)|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$

_____ 8.

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

_____ 9.

If $f(x) = \sqrt{2x}$, then $f'(2) =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

_____ 10.

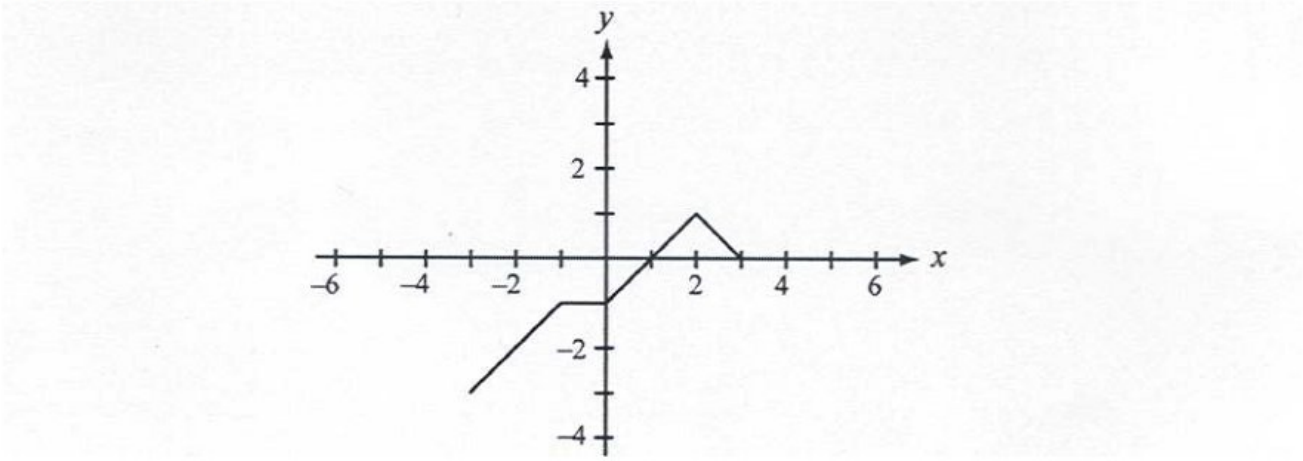
A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

Free Response

11. 1970-AB2 (No Calculator)

A function f is defined on the closed interval from -3 to 3 and has the graph shown below.



a. Sketch the entire graph of $y = |f(x)|$.

b. Sketch the entire graph of $y = f(|x|)$.

c. Sketch the entire graph of $y = f(-x)$.

d. Sketch the entire graph of $y = f\left(\frac{1}{2}x\right)$.

e. Sketch the entire graph of $y = f(x-1)$.

12. 1971-AB2 (No Calculator)

Let R be the region in the first quadrant that lies below both of the curves $y = 3x^2$ and $y = \frac{3}{x}$ and to the left of the line x = k, where k > 1.

a. Find the area of R as a function of k.

b. When the area of R is 7, what is the value of k?

c. If the area of R is increasing at the constant rate of 5 square units per second at what rate is k increasing when k = 15?