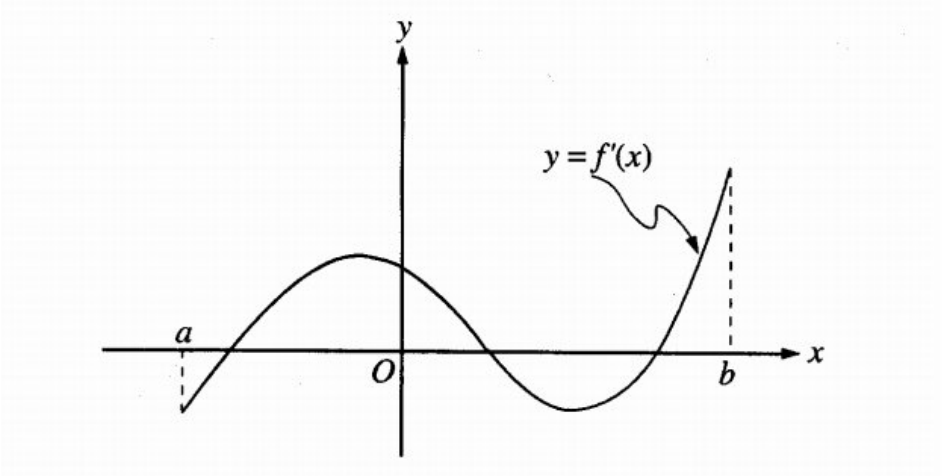
AP Calculus AB
Review 22, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

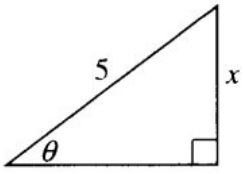
(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

_____ 2.

A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

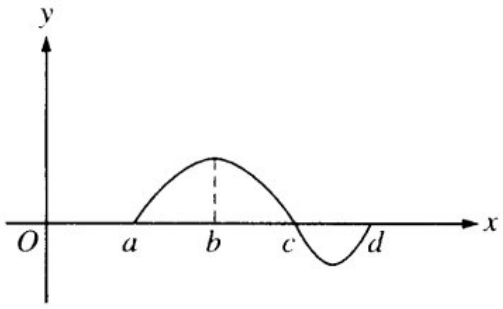
_____ 3.



In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

_____ 4.



The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.

_____ 5.

The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

_____ 6.

When the region enclosed by the graphs of $y = x$ and $y = 4x - x^2$ is revolved about the y-axis, the volume of the solid generated is given by

(A) $\pi \int_0^3 \left(x^3 - 3x^2\right) dx$

(B) $\pi \int_0^3 \left(x^2 - \left(4x - x^2\right)^2\right) dx$

(C) $\pi \int_0^3 \left(3x - x^2\right)^2 dx$

(D) $2\pi \int_0^3 \left(x^3 - 3x^2\right) dx$

(E) $2\pi \int_0^3 \left(3x^2 - x^3\right) dx$

_____ 7.

$\lim_{h \to 0} \dfrac{\ln(e+h) - 1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \dfrac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x + e)$

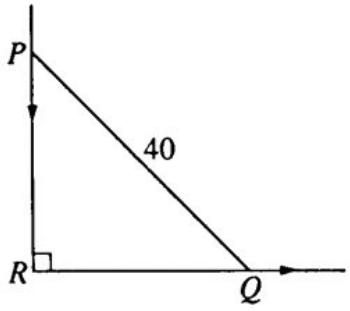
(E) $f'(0)$, where $f(x) = \ln x$

_____ 8.

Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\dfrac{30}{\pi^2}$ cm (E) $\dfrac{10}{\pi}$ cm

_____ 9.



In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

_____ 10.

Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

11. 2002—AB3 (Calculator Permitted)

An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time $t = 4$?

(b) Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

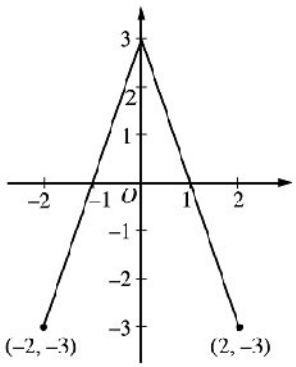
Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

(c) What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

(d) What is the position of the object at time $t = 4$?

12. 2002—AB4 (No Calculator)

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.



Graph of f

(a) Find $g(1)$, $g'(1)$, and $g''(1)$.

(b) For what values of x in the open interval $(-2,2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2,2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2,2]$.