AP Calculus AB
Review 23, Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.
Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

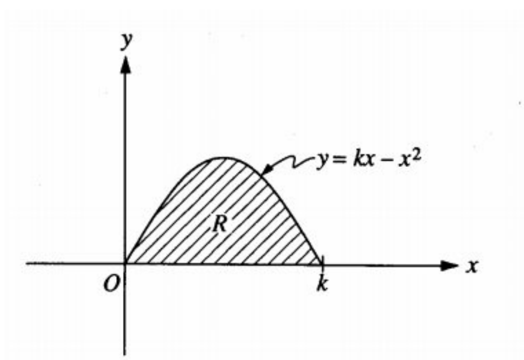
_____ 2.
If $0 \leq x \leq 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2 - 2t)\,dt \geq \int_2^x t\,dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

_____ 3.
If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

_____ 4.



The shaded region R, shown in the figure above, is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k?

(A) 1.51 (B) 2.09 (C) 2.49 (D) 4.18 (E) 4.77

_____ 5.
During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

_____ 6.

Let f and g be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$.

If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{f'(x)}{g'(x)}$

(C) $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$

(D) $\frac{f'(x)g(x) - f(x)g'(x)}{\left(f(x)\right)^2}$

(E) nonexistent

_____ 7.

If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b \left(f(x) + g(x)\right) dx = 4$

III. $\int_a^b \left(f(x)g(x)\right) dx = -5$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

_____ 8.

Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

_____ 9.

Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

(A) $f(0)=0$

(B) $f'(c)=\frac{4}{9}$ for at least one c between -3 and 6

(C) $-1 \le f(x) \le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

_____ 10.

If F and f are differentiable functions such that $F(x)=\int_0^x f(t)dt$, and if $F(a)=-2$ and $F(b)=-2$ where $a<b$, which of the following must be true?

(A) $f(x)=0$ for some x such that $a<x<b$.

(B) $f(x)>0$ for all x such that $a<x<b$.

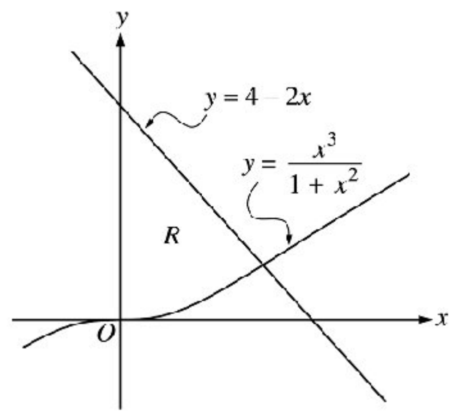
(C) $f(x)<0$ for all x such that $a<x<b$.

(D) $F(x)\le 0$ for all x such that $a<x<b$.

(E) $F(x)=0$ for some x such that $a<x<b$.

11. 2002—AB1B (Calculator Permitted)

Let R be the region bounded by the y-axis and the graphs of $y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

12. 2002—AB2B (Calculator Permitted)

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?