

AP Calculus AB

Review 24, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.

If $f(x) = 2x^2 + 1$, then $\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x^2}$ is

- (A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

_____ 2.

If p is a polynomial of degree n , $n > 0$, what is the degree of the polynomial $Q(x) = \int_0^x p(t) dt$?

- (A) 0 (B) 1 (C) $n - 1$ (D) n (E) $n + 1$

_____ 3.

If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

- (A) f is continuous for all real numbers.
(B) f has a minimum at $x = 0$.
(C) f is increasing for $x > 0$.
(D) $f'(x)$ exists for all x .
(E) $f''(x)$ is negative for $x > 0$.

_____ 4.

Which of the following functions are continuous at $x = 1$?

- I. $\ln x$
II. e^x
III. $\ln(e^x - 1)$

- (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 5.

If $\frac{dy}{dx} = x^2 y$, then y could be

- (A) $3 \ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

_____ 6.

The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

- (A) None (B) One (C) Two (D) Three (E) Four

_____ 7.

If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

- (A) $e^{\tan^2 x}$
(B) $\sec^2 x e^{\tan^2 x}$
(C) $\tan^2 x e^{\tan^2 x - 1}$
(D) $2 \tan x \sec^2 x e^{\tan^2 x}$
(E) $2 \tan x e^{\tan^2 x}$

_____ 8

The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

- (A) 0 (B) 1 (C) e (D) e^2 (E) $1 - e$

_____ 9.

The value of the derivative of $y = \frac{\sqrt[3]{x^2 + 8}}{\sqrt[4]{2x + 1}}$ at $x = 0$ is

- (A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

10.

A particle moves along the x -axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t > 0$ is $x(t) =$

(A) $-\frac{e^{-2t}}{2} + 3$

(B) $\frac{e^{-2t}}{4} + 4$

(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$

(E) $\frac{e^{-2t}}{4} + 3t + 4$

11. 2002—AB3B (Calculator Permitted)

A particle moves along the x -axis so that its velocity v at any time t , for $0 \leq t \leq 16$, is given by $v(t) = e^{2 \sin t} - 1$. At time $t = 0$, the particle is at the origin.

- On the axes provided, sketch the graph of $v(t)$ for $0 \leq t \leq 16$.
- During what intervals of time is the particle moving to the left? Give a reason for your answer.
- Find the total distance traveled by the particle from $t = 0$ to $t = 4$.
- Is there any time t , $0 < t \leq 16$, at which the particle returns to the origin? Justify your answer.

12. 2002—AB4B (No Calculator)

The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let

$$g(x) = 5 + \int_6^x f(t) dt \text{ for } -3 \leq x \leq 15.$$

- Find $g(6)$, $g'(6)$, and $g''(6)$.
- On what intervals is g decreasing? Justify your answer.
- On what intervals is the graph of g concave down? Justify your answer.
- Find a trapezoidal approximation of $\int_{-3}^{15} f(t) dt$ using six subintervals of length $\Delta t = 3$.

