

BC Review 11 No Calculator

Do all work on separate notebook paper

_____ 1.

The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \leq t \leq 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1} dt$

(B) $\int_0^1 \sqrt{t^2 + t} dt$

(C) $\int_0^1 \sqrt{t^4 + t^2} dt$

(D) $\frac{1}{2} \int_0^1 \sqrt{4 + t^4} dt$

(E) $\frac{1}{6} \int_0^1 t^2 \sqrt{4t^2 + 9} dt$

_____ 2.

If $\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

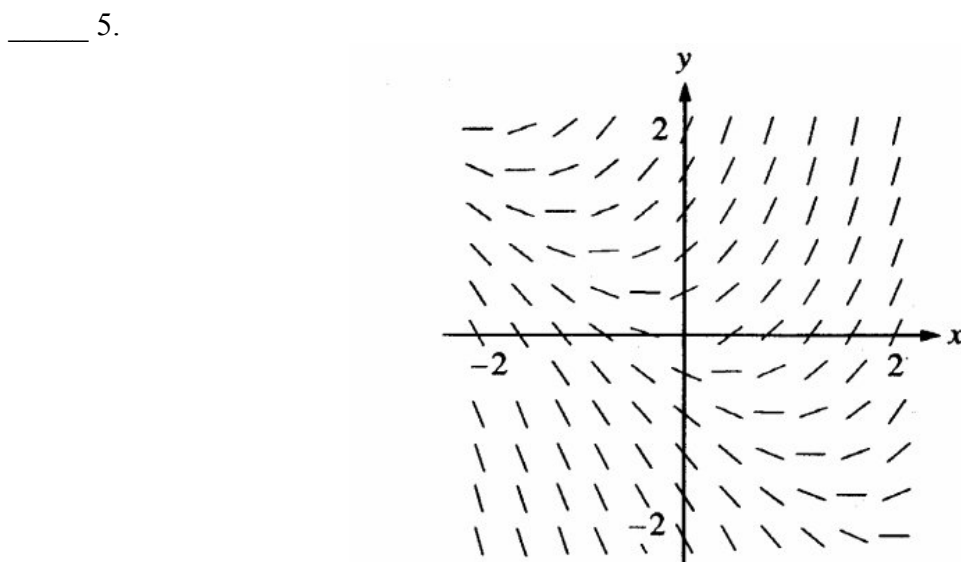
_____ 3.
 $\int_0^{\infty} x^2 e^{-x^3} dx$ is

- (A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

_____ 4.
 Let f be a function defined and continuous on the closed interval $[a, b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements must be true?

- I. $f'(c)$ exists.
 II. If $f'(c)$ exists, then $f'(c) = 0$.
 III. If $f''(c)$ exists, then $f''(c) \leq 0$.

- (A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only



Shown above is a slope field for which of the following differential equations?

- (A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

_____ 6.
 If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x , then $f'(1) =$

- (A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n^{n-1}$

_____ 7.

The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3,000$ and t is the time in years. What is $\lim_{t \rightarrow \infty} P(t)$?

- (A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

_____ 8.

$\lim_{x \rightarrow 1} \frac{\int_1^x e^{t^2} dt}{x^2 - 1}$ is

- (A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

_____ 9.

For what integer k , $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2

_____ 10.

If f is a vector-valued function defined by $f(t) = (e^{-t}, \cos t)$, then $f''(t) =$

- (A) $-e^{-t} + \sin t$ (B) $e^{-t} - \cos t$ (C) $(-e^{-t}, -\sin t)$
(D) $(e^{-t}, \cos t)$ (E) $(e^{-t}, -\cos t)$

Free Response

11. 2009—BC4 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

- (a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.
- (b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is -12 . Find the second-degree Taylor polynomial for f about $x = -1$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

12. 2009—AB/BC5 (No Calculator)

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \leq x \leq 13$.

(a) Estimate $f'(4)$. Show the work that leads to your answer.

(b) Evaluate $\int_2^{13} (3 - 5f'(x)) dx$. Show the work that leads to your answer.

(c) Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x) dx$. Show the work that leads to your answer.

(d) Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \leq x \leq 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \leq 4$. Use the secant line for the graph of f on $5 \leq x \leq 8$ to show that $f(7) \geq \frac{4}{3}$.
