BC Review 15 No Calculator
Do all work on separate notebook paper

_____ 1.

The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

_____ 2.

If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 3.

Let f and g have continuous first and second derivatives everywhere. If $f(x) \le g(x)$ for all real x, which of the following must be true?

I. $f'(x) \le g'(x)$ for all real x
II. $f''(x) \le g''(x)$ for all real x
III. $\int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

_____ 4.

$\int_1^\infty \frac{x}{(1+x^2)^2}\,dx$ is

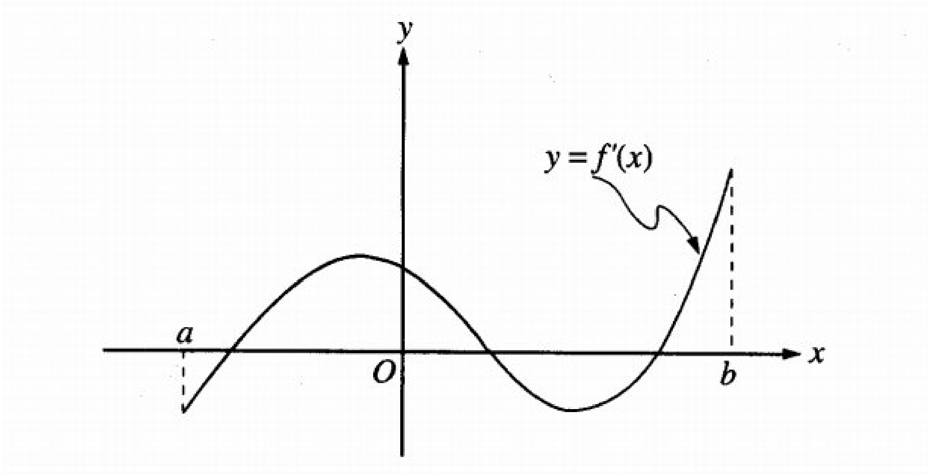
(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

_____ 5.

If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

_____ 6.



The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

_____ 7.

A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

_____ 8.

The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1,024}+\ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

_____ 9.

The length of the path described by the parametric equations $x=\cos^3 t$ and $y=\sin^3 t$, for $0\le t\le\frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}}\sqrt{3\cos^2 t+3\sin^2 t}\,dt$

(B) $\int_0^{\frac{\pi}{2}}\sqrt{-3\cos^2 t\sin t+3\sin^2 t\cos t}\,dt$

(C) $\int_0^{\frac{\pi}{2}}\sqrt{9\cos^4 t+9\sin^4 t}\,dt$

(D) $\int_0^{\frac{\pi}{2}}\sqrt{9\cos^4 t\sin^2 t+9\sin^4 t\cos^2 t}\,dt$

(E) $\int_0^{\frac{\pi}{2}}\sqrt{\cos^6 t+\sin^6 t}\,dt$

_____ 10.

$\lim\limits_{h\to 0}\frac{e^h-1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

Free Response

11. 2008—BC5

The derivative of a function f is given by $f'(x) = (x - 3)e^x$ for $x > 0$, and $f(1) = 7$.

(a) The function f has a critical point at $x = 3$. At this point, does f have a relative minimum, a relative maximum, or neither? Justify your answer.

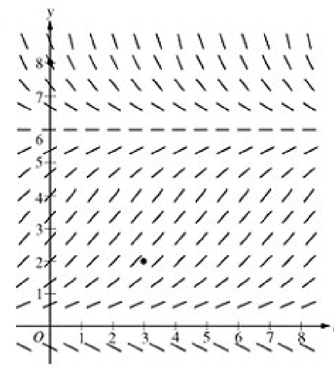
(b) On what intervals, if any, is the graph of f both decreasing and concave up? Explain your reasoning.

(c) Find the value of $f(3)$.

12. 2008—BC6

Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.
(Note: Use the axes provided in the exam booklet.)



(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?