AP CALCULUS
FORMULA LIST

Definition of e: $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Absolute value: $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

Definition of the derivative:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad \text{(Alternative form)}$$

Definition of continuity: f is continuous at c iff

1) $f(c)$ is defined;
2) $\lim_{x\to c} f(x)$ exists;
3) $\lim_{x\to c} f(x) = f(c)$.

Average rate of change of $f(x)$ on [a, b] $= \dfrac{f(b)-f(a)}{b-a}$

Rolle's Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there is at least one number c on (a, b) such that $f'(c) = 0$.

Mean Value Theorem: If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

Intermediate Value Theorem: If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\frac{d}{dx}[c] = 0$$

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$$\frac{d}{dx}[uv] = uv' + vu'$$

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

$$\frac{d}{dx}\left[f\left(g\left(x\right)\right)\right] = f'\left(g\left(x\right)\right)\cdot g'\left(x\right)$$

$$\frac{d}{dx}[\sin u] = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}[\cos u] = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}[\cot u] = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}[\sec u] = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}[\csc u] = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}[\log_a u] = \frac{1}{u \ln a}\frac{du}{dx}$$

$$\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$$

$$\frac{d}{dx}[a^u] = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}[\arcsin u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arccos u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arctan u] = \frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\text{arc}\cot u] = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\text{arc}\sec u] = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\frac{d}{dx}[\text{arc}\csc u] = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\left(f^{-1}\right)'\left(a\right) = \frac{1}{f'\left(f^{-1}\left(a\right)\right)}$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sin u \, du = -\cos u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

$$\int \frac{1}{u} \, du = \ln|u| + C$$

$\int \tan u \, du = -\ln|\cos u| + C$

$\int \cot u \, du = \ln|\sin u| + C$

$\int \sec u \, du = \ln|\sec u + \tan u| + C$

$\int \csc u \, du = -\ln|\csc u + \cot u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \frac{a^u}{\ln a} + C$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} arc\sec \frac{|u|}{a} + C$

Definition of a Critical Number:
Let f be defined at c. If $f'(c) = 0$ or if f' is undefined at c, then c is a critical number of f.

First Derivative Test:
Let c be a critical number of a function f that is continuous on an open interval I containing c. If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows.

1) If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a **relative minimum** of f.
2) If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a **relative maximum** of f.

Second Derivative Test:
Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1) If $f''(c) > 0$, then $f(c)$ is a relative minimum.
2) If $f''(c) < 0$, then $f(c)$ is a relative maximum.

Definition of Concavity:
Let f be differentiable on an open interval I. The graph of f is **concave upward** on I if f' is increasing on the interval and **concave downward** on I if f' is decreasing on the interval.

Test for Concavity:
Let f be a function whose second derivative exists on an open interval I.

1) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward in I.
2) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward in I.

Definition of an Inflection Point:
A function f has an inflection point at $(c, f(c))$

1) if $f''(c) = 0$ or $f''(c)$ does not exist and
2) if f'' changes sign at $x = c$. (or if $f'(x)$ changes from increasing to decreasing or vice versa at $x = c$)

First Fundamental Theorem of Calculus: $\int_a^b f'(x)\,dx = f(b) - f(a)$

Second Fundamental Theorem of Calculus: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Chain Rule Version: $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$

Average value of $f(x)$ on $[a, b]$: $f_{AVE} = \frac{1}{b-a}\int_a^b f(x)dx$

Volume around a horizontal axis by discs: $V = \pi\int_a^b [r(x)]^2\,dx$

Volume around a horizontal axis by washers: $V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)dx$

Volume by cross sections taken perpendicular to the *x*-axis: $V = \int_a^b A(x)dx$

If an object moves along a straight line with position function $s(t)$, then its

Velocity is $v(t) = s'(t)$

Speed = $|v(t)|$

Acceleration is $a(t) = v'(t) = s''(t)$

Displacement (change in position) from $x = a$ to $x = b$ is Displacement = $\int_a^b v(t)\,dt$

Total Distance traveled from $x = a$ to $x = b$ is Total Distance = $\int_a^b |v(t)\,dt|$

or Total Distance = $\left|\int_a^c v(t)\,dt\right| + \left|\int_c^b v(t)\,dt\right|$, where $v(t)$ changes sign at $x = c$.

CALCULUS BC ONLY

Integration by parts: $\int u\,dv = uv - \int v\,du$

Length of arc for functions: $s = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$

If an object moves along a curve, its

Position vector = $(x(t),\ y(t))$

Velocity vector = $(x'(t),\ y'(t))$

Acceleration vector = $(x''(t),\ y''(t))$

Speed (or magnitude of velocity vector) = $|v(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

Distance traveled from $t = a$ to $t = b$ (or length of arc) is $s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

In polar curves, $x = r\cos\theta$ and $y = r\sin\theta$

Slope of polar curve: $\dfrac{dy}{dx} = \dfrac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}$

Area inside a polar curve: $A = \dfrac{1}{2}\int_a^b r^2\, d\theta$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$