Korpi's Last-Minute AP Things Not to Forget to Remember

- Integration by *u*-substitution
  - When your "inside" function is linear, and it's derivative is "off" by more than a constant.
  - Let $u = \text{"inside" function}$ and change all variables from *x*, *dx* to *u*, *du*.
  - **Example**: $\int_0^1 \frac{2x}{\sqrt{x+2}}dx$

- Logarithmic Differentiation (LOG DIFF)
  - When you are taking the derivative of a variable function raised to a power of a variable function.
  - Take the natural log of both sides, differentiate implicitly, then resolve for *y*.
  - **Example**: $\frac{d}{dx}\left[x^{\sin x}\right] =$

- Derivatives of Trig/inverse Trig functions
  - $\frac{d}{dx}\arcsin x =$ $\qquad \frac{d}{dx}\arctan x =$ $\qquad \frac{d}{dx}\operatorname{arc}\sec x =$
  - $\frac{d}{dx}\arccos x =$ $\qquad \frac{d}{dx}\operatorname{arc}\cot x =$ $\qquad \frac{d}{dx}\operatorname{arc}\csc x =$
  - $\frac{d}{dx}\sin x =$ $\qquad \frac{d}{dx}\tan x =$ $\qquad \frac{d}{dx}\sec x =$
  - $\frac{d}{dx}\cos x =$ $\qquad \frac{d}{dx}\cot x =$ $\qquad \frac{d}{dx}\csc x =$
  - Example: $\frac{d}{dx}\cos^{-1}\left(\tan\left(2x^2\right)\right) =$

- Integral of Trig functions
  - $\int \sin x\,dx =$ $\int \cos x\,dx =$ $\int \tan x\,dx =$ $\int \cot x\,dx =$
  - $\int \sec x\,dx =$ $\int \csc x\,dx =$ $\int \sec^2 x\,dx =$ $\int \sec x \tan x\,dx =$
  - **Examples**: $\int \tan^2 x\,dx =$ $\int \frac{\cot\left(\sqrt{x}\right)}{\sqrt{x}}dx =$

- Finding extrema vs. finding the location of extrema
  - An extreme value is a *y*-value. It occurs at an *x*-value
  - **Example**: Find the maximum value of $f\left(x\right)=x^2-5$ on $\left[-1,2\right]$

- Finding slopes of inverse functions
  - Inverse functions, at corresponding points, have reciprocal slopes.
  - If $f\left(g\left(x\right)\right)=x=g\left(f\left(x\right)\right)$, then $f\left(x\right)$ and $g\left(x\right)$ are inverses
  - $g\left(a\right)=b$ implies $f\left(b\right)=a$
  - $g'\left(a\right)=\frac{1}{f'\left(b\right)}$
  - **Example**: If $f\left(x\right)=g^{-1}\left(x\right)$ and $f\left(x\right)=2x^2+3x-1$ and if $g\left(-2\right)=-1$, find $g'\left(-2\right)$.

- Finding the slope of a normal line to a function, $f(x)$, at a point $x = a$
  - Normal lines are perpendicular to tangent lines at a point.
  - The normal slope, *n*, is the opposite, reciprocal of the tangent slope.
  - $n = \dfrac{-1}{f'(a)}$
  - **Example**: Find the equation of the normal line to the graph of $y = e^{2x}$ at $x = \ln 2$

- Squiggle Alert when the words "approximate" or "estimate" are used in the question
  - Explicitly stated approximations must have an approximation symbol, $\approx$, rather than an equal sign.
  - This happens with tangent line approximations, numeric methods of integration, linearization, Euler's Method (BC), and Taylor Polynomials (BC).
  - **Example**: If *f* is differentiable, and if $f(3) = 2$ and $f(5.5) = 5$, approximate $f'(4.1)$. Show the work that leads to your answer.

- Average value vs. average rate of change
  - Average value is the average ***y***-value for whatever is being measured on the *y*-axis.
  - Average value is "integral over width"
  - Average value $= \dfrac{\int_a^b f(x)\,dx}{b-a} = \dfrac{1}{b-a}\int_a^b f(x)\,dx$
  - Average rate of change is the change in *y* over the change in *x*—the slope of the secant line
  - Average rate of change $= \dfrac{f(b) - f(a)}{b-a}$
  - **Example**: If $W(t)$ is the rate at which rain falls on a roof of a house for $0 \le t \le 3$, where *W* is measured in cm/hr and *t* is measured in hours. Explain the meaning, with correct units, of $\frac{1}{3}\int_0^3 W(t)\,dt$ in the context of rainfall.

- The misuse of equality a.k.a. mathematically prevarication
    - If you use an equal sign, the expressions you are equating better be equal, or you will lose a point.
    - **Example**: If $f(x)=2x^2-1$, find $f'(1)$
        - WRONG: $f'(x)=4x=4$
        - CORRECT: $f'(x)=4x$, $f'(1)=4(1)=4$
    - **Example**: To the nearest whole number, find $\int_0^2 e^x\,dx$
        - WRONG: $\int_0^2 e^x\,dx=6.389=6$
        - CORRECT: $\int_0^2 e^x\,dx=6.389\approx 6$ OR $\int_0^2 e^x\,dx=6$

- Be sure to include units in all final numeric answers AND any written explanation of this answer (including both independent AND dependent variables.
    - Units can cost you an entire point if you omit them or use the wrong ones.
    - **Example**: $w(t)$ is the temperature of water in a jug in a refrigerator, in $^\circ F$, where *t* is in minutes.

      In the context of the problem, explain the meaning of (a) $w(5)=-2.1$ (b) $\frac{1}{5}\int_0^5 w(t)\,dt=44$

- Speed increasing or decreasing vs. Velocity increasing or decreasing
    - If at $t=c$, $v(c)>0$ AND $a(c)>0$ or $v(c)<0$ AND $a(c)<0$, then speed is increasing at $t=c$.
    - If at $t=c$, $v(c)<0$ AND $a(c)>0$ or $v(c)>0$ AND $a(c)<0$, then speed is decreasing at $t=c$.
    - If at $t=c$, $v'(c)=a(c)>0$, then velocity is increasing at $t=c$.
    - If at $t=c$, $v'(c)=a(c)<0$, then velocity is decreasing at $t=c$.
    - If the graph of $v(t)$ moves TOWARD the *t* axis, speed is decreasing.
    - If the graph of $v(t)$ moves AWAY FROM the *t* axis, speed is increasing.
    - **Example**: If a particle moves along the *x*-axis such that for $t\ge 0$, its position is give by , $x(t)=\frac{1}{3}t^3-4t^2+15t-7$, at $t=4.5$, is the speed of the particle increasing or decreasing? At this time is the velocity of the particle increasing or decreasing? Justify your answers.

- IVT (The Intermediate Value Theorem)
  - If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ takes on all *y*-values between $f(a)$ and $f(b)$.
  - **Example**: If $f(x)$ is a differentiable function such that $f(-1)=-3$ and $f(4)=\frac{5}{6}$, explain why $f(x)$ must have a root on the interval $(-1,4)$.

- EVT (The Extreme Value Theorem)
  - If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both a Maximum and Minimum on the CLOSED interval $[a,b]$.
  - **Example**: If a particle moves along the *x*-axis such that its position is give by, $x(t)=t^2-3t+2$, on the interval $0\le t\le 2$, at what time, *t*, is the particle farthest left? Farthest right?

- MVT (The Mean Value Theorem)
  - If $f(x)$ is continuous on a closed interval $[a,b]$, and differentiable on the open interval (a,b), then there is an $x=c$ on the OPEN interval (a,b), where the slope of the tangent line (instantaneous rate of change/derivative) equals the slope of the secant line (average rate of change).
  - USED TO SHOW THAT A DERIVATIVE EXISTS ON AN INTERVAL
  - Set it up as $f'(x)=\frac{f(b)-f(a)}{b-a}$, then solve for *x*, then make sure *x* is in the OPEN interval!
  - **Example**: If $f(x)=x^3+x-4$, on the interval $-1\le x\le 2$, find the value of *c* guaranteed by the Mean Value Theorem.
  - **Example:** If $f'(x)$ is a differentiable function for all *x*, and if $f'(5)=-2$ and $f'(7)=4$, explain why there must be a *c*, $5<c<7$ such that $f''(c)=3$.

- Geometric formulas to remember
  - Volume of a Sphere: $V = \frac{4}{3}\pi r^3$
  - Surface area of a Sphere: $A = 4\pi r^2$
  - Volume of a Cone: $V = \frac{\pi}{3} r^2 h$
  - Volume of a Cylinder: $V = \pi r^2 h$
  - Surface area of a Cylinder: $A = 2\pi r^2 + 2\pi rh$
  - Equilateral Triangle: $A = \frac{\sqrt{3}}{4} s^2$
  - Trapezoid: $A = \frac{1}{2}\Delta x\left(y_1 + y_2\right)$
  - Rectangle: $A = h \cdot w$

- Justifying relative extrema using the First Derivative test and Second Derivative Test
  - First Derivative Test (at a critical point, $\left(c, f\left(c\right)\right)$)
    - "Since $f'\left(c\right) = 0$ (or $f'\left(c\right) = DNE$), and since $f'\left(x\right)$ changes from positive to negative at $x = c$, $f\left(x\right)$ has a Relative (local) Maximum at $x = c$."
    - "Since $f'\left(c\right) = 0$ (or $f'\left(c\right) = DNE$), and since $f'\left(x\right)$ changes from negative to positive at $x = c$, $f\left(x\right)$ has a Relative (local) Minimum at $x = c$."
  - Second Derivative Test (at a critical value$\left(c, f\left(c\right)\right)$)
    - "Since $f'\left(c\right) = 0$ (or $f'\left(c\right) = DNE$), and since $f''\left(c\right) < 0$, $f\left(x\right)$ has a Relative (local) Maximum at $x = c$."
    - "Since $f'\left(c\right) = 0$ (or $f'\left(c\right) = DNE$), and since $f''\left(c\right) > 0$, $f\left(x\right)$ has a Relative (local) Minimum at $x = c$."

- Justifying an inflection point at a p.i.v. (possible inflection value)
  - If $f\left(c\right)$ is defined, and either $f''\left(c\right) = 0$ or $f''\left(c\right) = DNE$, then $f\left(x\right)$ has an inflection point at $\left(c, f\left(c\right)\right)$ if f'' changes from positive to negative at $x = c$ or negative to positive at $x = c$.
  - **Example**: A continuous function $f\left(x\right)$ has a second derivative $f''\left(x\right) = \frac{\left|x-3\right|}{x-3}$. Determine if $f\left(x\right)$ has an inflection value or not at $x = 3$. Justify.

- Cross-sectional volume magic numbers
  - Squares: Equilateral Triangles: Semicircles: Rectangles with height *n* times the base
  - Quarter Circles: Isos Rt Triangle, Leg in Base: Isos Rt Triangle, Hypot in Base:
- Inverse Trig Integral formulas
  - $\int \frac{du}{a^2+u^2} =$ $\int \frac{du}{\sqrt{a^2-u^2}} =$ $\int \frac{du}{u\sqrt{u^2-a^2}} =$
  - **Examples**:

    $\int \frac{1}{x^2+5}dx =$ $\int \frac{5}{\sqrt{7-9x^2}}dx =$ $\int \frac{1}{\sqrt{e^{2x}-2}}dx =$
- Convergence Tests (used also to determine endpoints of intervals of convergence)
  - *n*th term test for divergence
  - Geometric series *p*-series Direct/Limit Comparison Test
  - Integral Test Ratio Test Alternating Series Test/Error
- Pen or Pencil on the exam???
- 3-decimal accuracy (round or truncate). Store non-exact answers needed for future calculations and LABEL THEM ON YOUR PAPER. Avoid duplicate letters. NEVER use an approximate answer to calculate a subsequent value.

- Implicit Differentiation—your derivative will have both x and y in it.
  - You will have as many $\frac{dy}{dx}$'s in your derivative as you have y's in your equation.
  - Look to solve for y first, especially if your answer choices are in terms of x only and/or you are finding an actual value and are only given $x = a$.
  - When solving for $\frac{dy}{dx}$, if you ever end up with an answer like $\frac{dy}{dx} = \frac{a-b}{c-d}$, realize that this is equivalent to $\frac{dy}{dx} = \frac{b-a}{d-c}$.
  - When finding a second derivative (or higher order derivative) implicitly, if the instructions say "in terms of x and y," be sure to plug in your $\frac{dy}{dx}$ expression into your final answer.

- Parenthesis are your best friends—it's better to have them and not need them than to need them and not have them. This is especially true for:
  - $k\int_a^b f(x)\,dx = k\left[\left(f(b)\right) - \left(f(a)\right)\right]$ $\qquad$ $\int_a^b \left[f(x) + g(x)\right]dx$
  - $\pi\int_a^b \left[\left(R(x)\right)^2 - \left(r(x)\right)^2\right]dx$ $\qquad$ $\lim_{h\to 0} \frac{\left(\left(x+h\right)^2 - 2\left(x+h\right) + 1\right) - \left(x^2 - 2x\right) + 1}{h} =$
  - $5 - \cos^2 x = 5 - \left(1 - \sin^2 x\right)$ $\qquad$ $\frac{5}{2-\sqrt{x-3}} \cdot \frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}$

- Cusp Alert!
  - If you have a variable raised to a power that is between zero and one, you will have a continuous function that is not differentiable. **The root of the term with the alerted power will be a critical value of the function!**
  - **Example**: Find the critical values of $f(x) = x^{2/3} - x$

- Pronouns. Don't Use Pronouns. Don't be vague, ambiguous, or unclear either.
  - Don't say things like, " . . . since **it** changes from positive to negative . . .," "since **the graph** is increasing," or "**the function** changes signs **there**."
  - Be explicit. Say what you mean and mean what you say.

- Don't waste time erasing. If you draw a line through something, it becomes "invisible" to the AP graders. ~~Why So Serious?~~, ~~We Are Sparta.~~

- Don't draw a line through anything on a free response or erase anything unless you have the time and intention of replacing it with something else. Something is better than nothing. Never leave anything blank, whether it's a M.C. or F.R. question. You EARN points on this test, not lose points.

- Don't let a wrong answer on one part of a F.R. question keep you from getting credit on subsequent parts. Using your wrong answer correctly, making up a reasonable equation to work with, or even attaching units to a number can get you points. NEVER give up. NEVER surrender.

- You don't need to simplify your numeric answers on the free response (unless the instructions explicitly tell you to approximate it to 3 decimals or ask you to "show" that a number equals a given number), but you MUST indicate your numeric methods to get credit. This especially goes for integral approximations from a table of values, difference quotients, and using areas from a graph to approximate integrals.

- Breath, Relax, Smile, and get that 5!