1. D
2. E
3. D
4. B
5. A
6. C
7. E
8. B
9. D
10. C

13

AB Review 13 No Calculator Permitted

1.

$$\int_0^1 e^{-4x}\,dx =$$

A) $\dfrac{-e^{-4}}{4}$ B) $-4e^{-4}$ C) $e^{-4}-1$ D) $\dfrac{1}{4}-\dfrac{e^{-4}}{4}$ E) $4-4e^{-4}$

2.

For $x \geq 0$, the horizontal line y=2 is an asymptote for the graph of the function f. Which of the following statements must be true?

A) $f(0)=2$

B) $f(x)\neq 2$ for all $x \geq 0$

C) $f(2)$ is undefined

D) $\lim\limits_{x\to 2} f(x)=\infty$

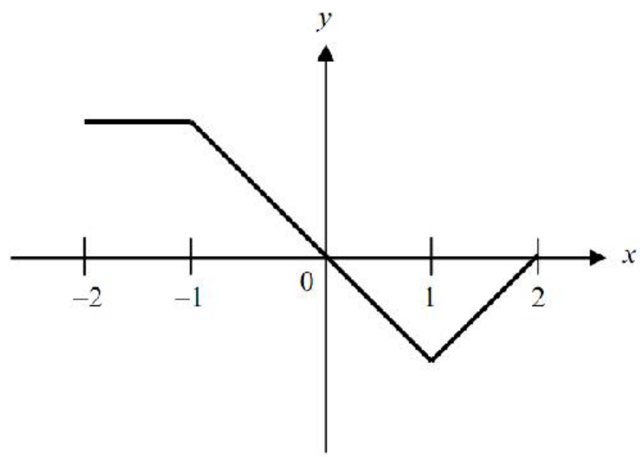
E) $\lim\limits_{x\to\infty} f(x)=2$

3.

$$\int_0^{\frac{\pi}{4}} \sin(x)dx =$$

A) $-\frac{\sqrt{2}}{2}$ B) $\frac{\sqrt{2}}{2}$ C) $-\frac{\sqrt{2}}{2}-1$ D) $-\frac{\sqrt{2}}{2}+1$ E) $\frac{\sqrt{2}}{2}-1$

4.



Graph of f'

The graph of f' , the derivative of the function f , is shown above. Which of the following statements is true about f ?

A) f is decreasing for $-1 \leq x \leq 1$.

B) f is increasing for $-2 \leq x \leq 0$.

C) f is increasing for $1 \leq x \leq 2$.

D) f has a local minimum at x = 0.

E) f is not differentiable at x = -1 and x = 1.

5.

If $f(x)=\ln\left(x+4+e^{-3x}\right)$, then $f'(0)$ is

A) $-\frac{2}{5}$ B) $\frac{1}{5}$ C) $\frac{1}{4}$ D) $\frac{2}{5}$ E) nonexistent

6.

Using the substitution $u=2x+1$, $\int_0^2 \sqrt{2x+1}dx$ is equivalent to

A) $\frac{1}{2}\int_{-1/2}^{1/2}\sqrt{u}du$ B) $\frac{1}{2}\int_0^2\sqrt{u}du$ C) $\frac{1}{2}\int_1^5\sqrt{u}du$ D) $\int_0^2\sqrt{u}du$ E) $\int_1^5\sqrt{u}du$

7.

The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?

A) $V(t)=k\sqrt{t}$

B) $V(t)=k\sqrt{V}$

C) $\frac{dV}{dt}=k\sqrt{t}$

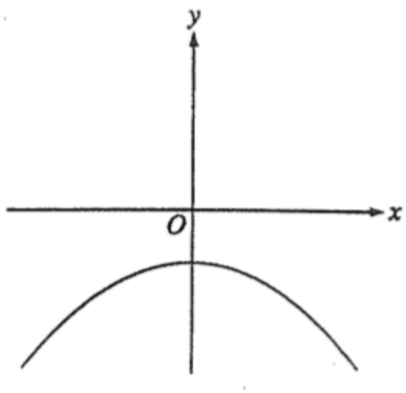
D) $\frac{dV}{dt}=\frac{k}{\sqrt{V}}$

E) $\frac{dV}{dt}=k\sqrt{V}$

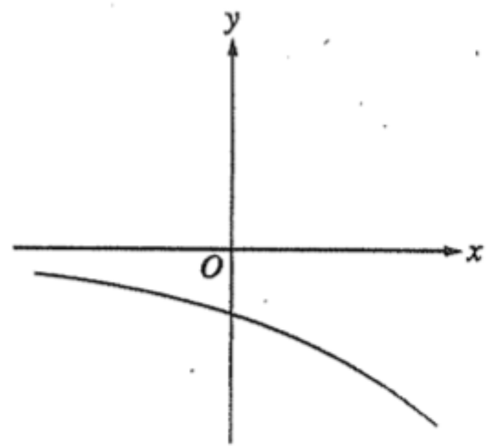
8.

The function f has the property that $f(x)$, $f'(x)$, $f''(x)$ and are negative for all real values x. Which of the following could be the graph of f ?

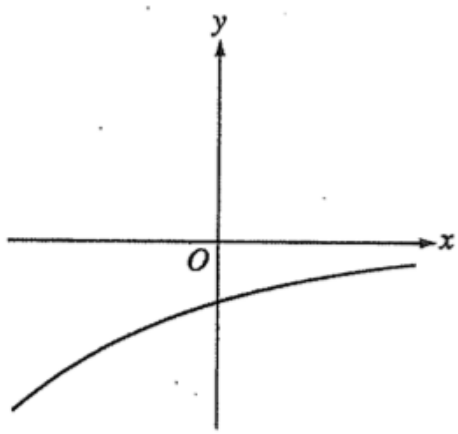
(A)



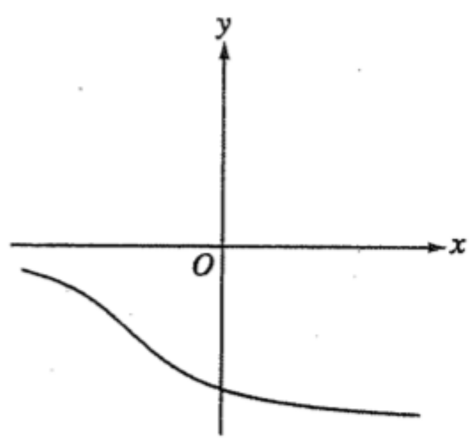
(B)



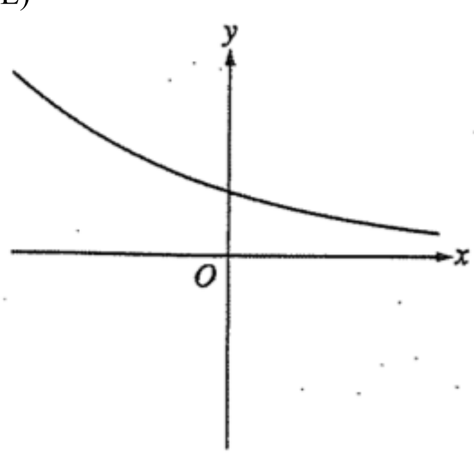
(C)



(D)



(E)



9.

Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

A) $(-\infty, -1]$ only

B) $(-\infty, 0)$

C) $[-1, 0)$ only

D) $(0, \sqrt[3]{2}]$

E) $[\sqrt[3]{2}, \infty)$

10.

If the line tangent to the graph of the function f at the point (1,7) passes through the point (-2, -2), then $f'(1)$ is

A)-5 B) 1 C) 3 D) 7 E) undefined

11. (2013, AB-3)

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)\,dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \dfrac{C(4) - C(3)}{4 - 3} = \dfrac{12.8 - 11.2}{1} = 1.6$ ounces/min

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval)

$\dfrac{C(4) - C(2)}{4 - 2} = \dfrac{12.8 - 8.8}{2} = 2$

Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.

$2 : \begin{cases} 1 : \dfrac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\dfrac{1}{6}\int_0^6 C(t)\,dt \approx \dfrac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$

$= \dfrac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$

$= \dfrac{1}{6}(60.6) = 10.1$ ounces

$\dfrac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.

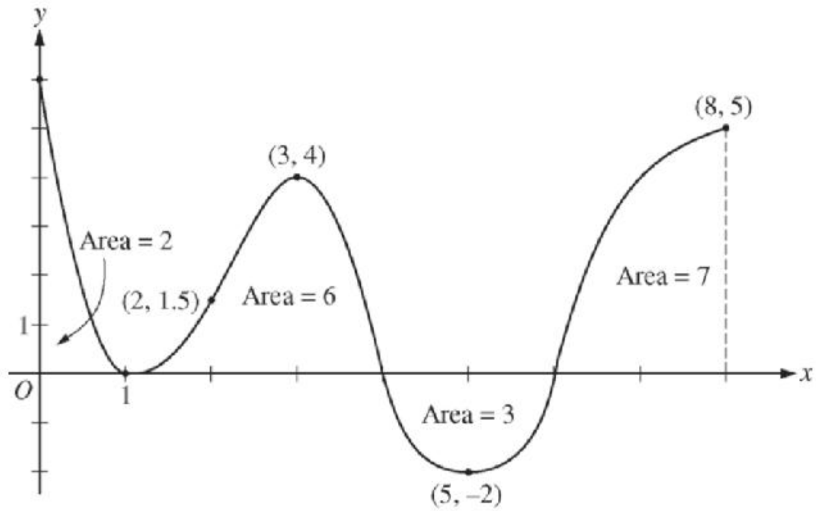
$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min

$2 : \begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

12. (2013, AB-4)



The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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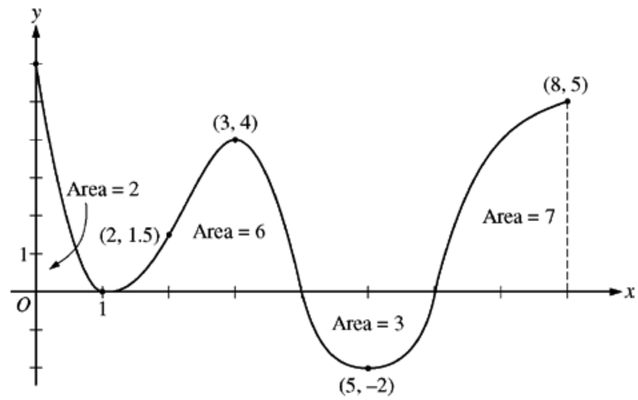
Question 4

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.



Graph of f'

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

(a) $x = 6$ is the only critical point at which f' changes sign from negative to positive. Therefore, f has a local minimum at $x = 6$.

1 : answer with justification

(b) From part (a), the absolute minimum occurs either at $x = 6$ or at an endpoint.

$$f(0) = f(8) + \int_8^0 f'(x)\,dx = f(8) - \int_0^8 f'(x)\,dx = 4 - 12 = -8$$

$$f(6) = f(8) + \int_8^6 f'(x)\,dx = f(8) - \int_6^8 f'(x)\,dx = 4 - 7 = -3$$

$$f(8) = 4$$

The absolute minimum value of f on the closed interval $[0, 8]$ is -8.

3 : { 1 : considers $x = 0$ and $x = 6$; 1 : answer; 1 : justification }

(c) The graph of f is concave down and increasing on $0 < x < 1$ and $3 < x < 4$, because f' is decreasing and positive on these intervals.

2 : { 1 : answer; 1 : explanation }

(d) $g'(x) = 3[f(x)]^2 \cdot f'(x)$

$$g'(3) = 3[f(3)]^2 \cdot f'(3) = 3\left(-\frac{5}{2}\right)^2 \cdot 4 = 75$$

3 : { 2 : $g'(x)$; 1 : answer }