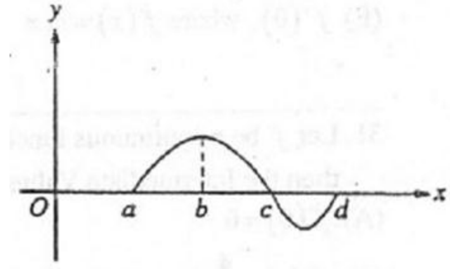
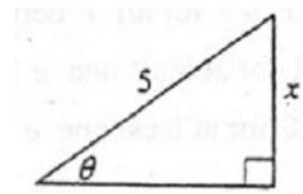
BC Review 03, No calculator.



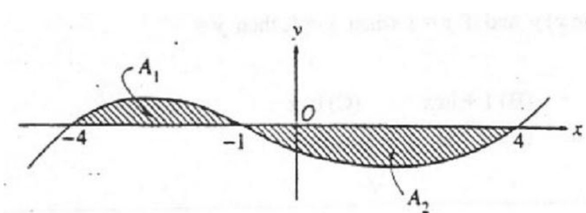
1. The graph of f is shown in the figure on the right. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

   (A) a (B) b (C) c (D) d (E) It cannot be determined from the information given



2. In the triangle shown on the right, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing, in units per minute, when $x=3$ units?

   (A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12



3. The graph of $y=f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2, $\int_{-4}^{4} f(x)\,dx-2\int_{-1}^{4} f(x)\,dx=$

   (A) A_1 (B) A_1-A_2 (C) $2A_1-A_2$ (D) A_1+A_2 (E) A_1+2A_2

4. $\lim_{t\to\infty}\left(3t^2\sin^2\left(\frac{2}{t}\right)\right)=$

(A) 18 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) 12 (E) $\frac{4}{3}$

5. $\int_1^4 \frac{dx}{\sqrt{16-x^2}}=$

(A) $\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (B) $-\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (C) $\arcsin\left(\frac{1}{4}\right)-\frac{\pi}{2}$

(D) $-4\arcsin\left(\frac{1}{4}\right)+\frac{\pi}{2}$ (E) $4\arcsin\left(\frac{1}{4}\right)-\frac{\pi}{2}$

6. Find the radius of convergence for the series $\sum_{k=1}^{\infty}\frac{4^{k+2}x^k}{k+1}$

(A) 1 (B) 0 (C) $\frac{1}{4}$ (D) 4 (E) The series diverges for all x

7. The position of a particle moving along the x-axis at time t is given by $x(t)=\sin^2(4\pi t)$. At which of the following values of t will the particle change direction?

I. $t=\frac{1}{8}$

II. $t=\frac{1}{6}$

III. $t=1$

IV. $t=2$

(A) II, III, and IV (B) I and II (C) I, II, and III (D) III and IV (E) I, III, and IV

8. Determine $\frac{dy}{dt}$ given that $y=-3x^2+4x$ and $x=\cos t$.

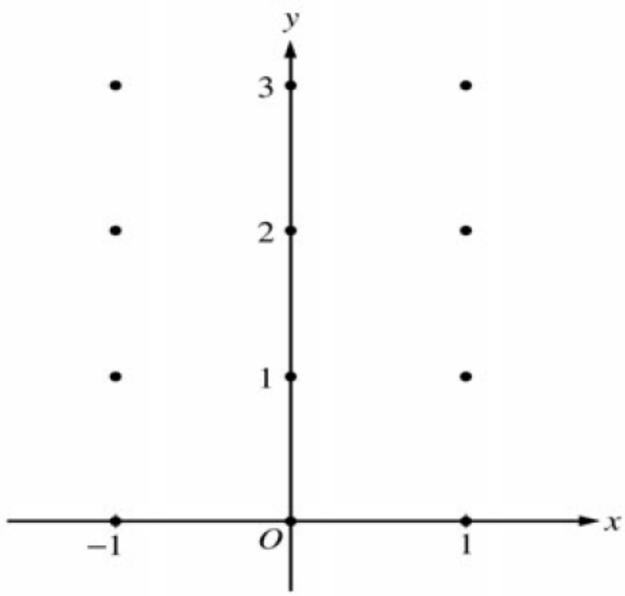
(A) $6\cos t-4$ (B) $-2\cos t\sin t$ (C) $-(-6\cos t+4)\sin t$ (D) $2\sin t$ (E) $-(-6\cos t+4)\cos t$

9. The function $f(x)=2x^2+4e^{5x}$ has an inverse function $f^{-1}(x)$. Find the slope of the <u>normal</u> line to the graph of $f^{-1}(x)$ at $x=f(0)$.

(A) $16+20e^{20}$ (B) $\frac{1}{20}$ (C) $-\frac{1}{16+20e^{20}}$ (D) -20 (E) $-\frac{5}{4}$

10. A circle centered at $(0,-3)$ with a radius of 3 has a polar equation

(A) $r=-6\sin\theta-\cos\theta$ (B) $r=-3\sin\theta-3\cos\theta$ (C) $r=-3\csc\theta$ (D) $r=-6\sin\theta$ (E) $r=-6\cos\theta$



11. (2004, AB-6) Consider the differential equation given by $\frac{dy}{dx} = x^2(y - 1)$.

(a) On the axes provided above, sketch a slope field for the given differential equation at the 12 points indicated.

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

12. (1999, AB-4) Suppose that the function f has a continuous second derivative for all x, and that $f(0)=2$, $f'(0)=-3$, and $f''(0)=0$. Let g be a function whose derivative is given by $g'(x)=e^{-2x}\left(3f(x)+2f'(x)\right)$ for all x.

(a) Write an equation of the tangent line to the graph of f at the point where $x=0$.

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x=0$? Explain your answer.

(c) Given that $g(0)=4$, write an equation of the line tangent to the graph of g at the point where $x=0$.

(d) Show that $g''(x)=e^{-2x}\left(-6f(x)-f'(x)+2f''(x)\right)$. Does g have a local maximum at $x=0$? Justify your answer.