BC Review 07, No Calculator Permitted, unless specified to the contrary.

1. (Calculator Permitted) Let f be the function given by $f(x)=3e^{2x}$ and let g be the function given by $g(x)=6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

   (A) -0.701 (B) -0.567 (C) -0.391 (D) -0.302 (E) -0.258

2. The radius of a circle is decreasing at a constant rate of 0.1 centimeters per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

   (A) $-(0.2)\pi C$ (B) $-(0.1)C$ (C) $-\frac{(0.1)C}{2\pi}$ (D) $(0.1)^2 C$ (E) $(0.1)^2 \pi C$

3. (Calculator Permitted) The first derivative of a function f is given by $f'(x)=\frac{\cos^2 x}{x}-\frac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

   (A) One (B) Three (C) Four (D) Five (E) Seven

4. Give the exact value of $\sum_{n=0}^{\infty}\frac{\cos(n\pi)4^n}{n!}$

   (A) e^4 (B) $\sin 4$ (C) $\cos 4$ (D) $-\sin 4$ (E) e^{-4}

5. Let f be the function given by $f(x)=|x|$. Which of the following statements about f are true?

I. f is continuous at $x=0$.

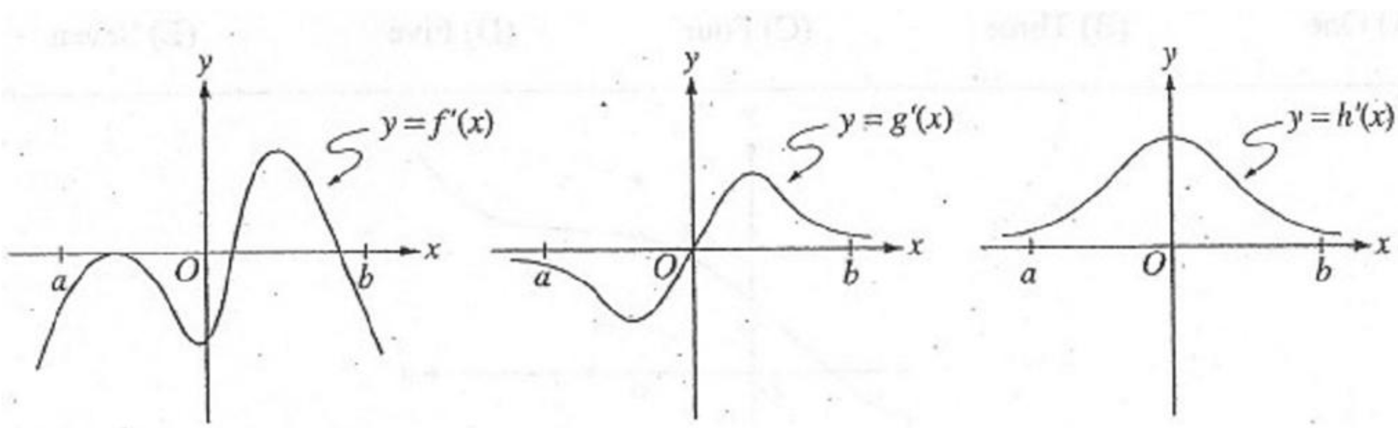
II. f is differentiable at $x=0$.

III. f has an absolute minimum at $x=0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

6. If f is a continuous function and if $F'(x)=f(x)$ for all real numbers x, then $\int_1^3 f(2x)dx=$

(A) $2F(3)-2F(1)$ (B) $\frac{1}{2}F(3)-\frac{1}{2}F(1)$ (C) $2F(6)-2F(2)$ (D) $F(6)-F(2)$ (E) $\frac{1}{2}F(6)-\frac{1}{2}F(2)$



7. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a<x<b$?

(A) f only (B) g only (C) h only (D) f and g only (E) f, g, and h

8. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

9. $\int_0^\infty \frac{dx}{16 + x^2} =$

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{2}$ (C) 2π (D) ∞ (E) 4π

10. $\lim_{x \to \infty} \left(1 + 7^x\right)^{1/x} =$

(A) 7 (B) 5 (C) ∞ (D) e^7 (E) 10

11. (2003, AB-6) Let f be the function defined by

$$f(x)=\begin{cases}\sqrt{x+1} & \text{for } 0\le x\le 3\\ 5-x & \text{for } 3<x\le 5\end{cases}$$

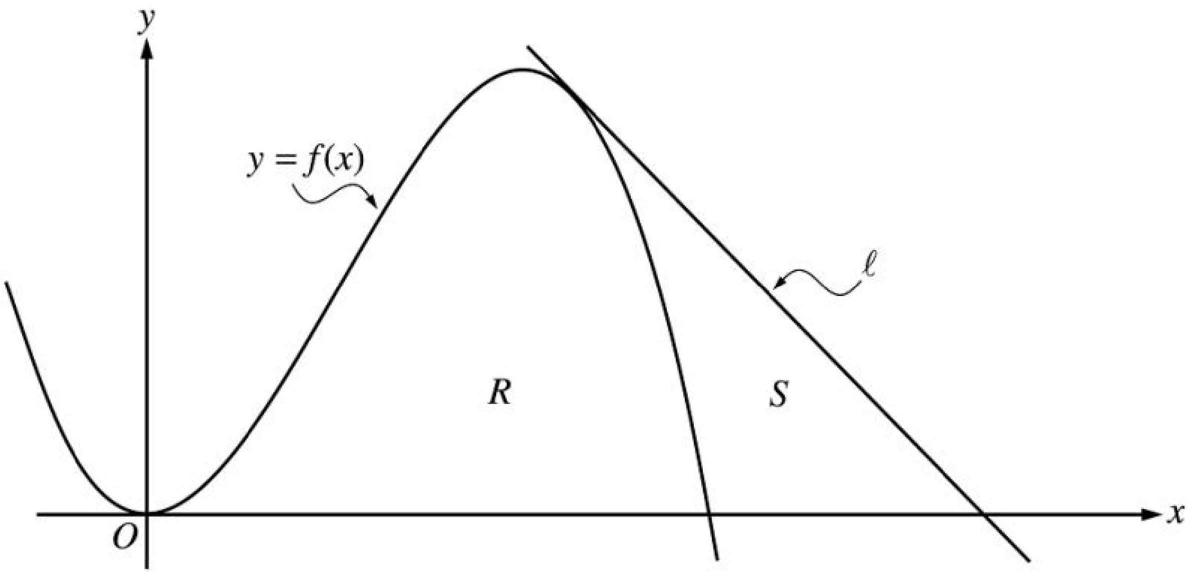
(a) Is f continuous at $x=3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0\le x\le 5$.

(c) Suppose the function g is defined by

$$g(x)=\begin{cases}k\sqrt{x+1} & \text{for } 0\le x\le 3\\ mx+2 & \text{for } 3<x\le 5\end{cases}$$

Where k and m are constants. If g is differentiable at $x=3$, what are the values of k and m?



12. (2003, AB/BC-1B) (Calculator Permitted) Let f be the function given by $f(x) = 4x^2 - x^3$, and let ℓ be the line $y = 18 - 3x$, where ℓ is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line ℓ, and the x-axis, as shown above.

(a) Show that ℓ is tangent to the graph of $y = f(x)$ at the point $x = 3$.

(b) Find the area of S.

(c) Find the volume of the solid generated when R is revolved about the x-axis.