BC Review FINAL, NO Calculator Permitted (unless stated otherwise)
Do all work on separate notebook paper

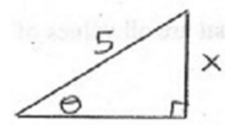
1. (a) $\int x\sin(2x)dx =$ (b) $\int \frac{dx}{x^2-6x+8} =$

2. Write an integral expression which gives the area of the region inside the polar curve $r = 4\cos\theta$ and outside $r = 2$.

3. Given $\frac{dy}{dx} = \frac{xy}{2}$. Let $f(x)$ be the particular solution to the given differential equation with initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of $0.1$, to approximate $f(0.2)$.

4. If $3xy + 2x = 1 - 2y$, then when $x = -1$, $\frac{dy}{dx} = ?$

5. If in the triangle at right, $\theta$ decreases by 4 rad/min, at what rate is $x$ changing in units/min when $x = 4$?




6. Write an integral equation which gives the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$ for $0 \le t \le \frac{\pi}{2}$.

7. If $f$ is a function such that $f'(x) = \sin\left(x^2\right)$, then the coefficient of $x^7$ in the Taylor series for $f(x)$ about $x = 0$ is?

8. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y = ?$

9. (a) $\int \tan^2 x dx =$ (b) $\int \sin^2 x dx =$

10. $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}\right)}{h} =$

11. The coefficient of $x^3$ in the Taylor series for $e^{2x}$ about $x = 0$ is?

12. What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converge?

13. Which of the following converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

14. The Taylor series about $x=5$ for a certain function $f$ converges to $f(x)$ for all $x$ in its interval of convergence. The $n$th derivative of $f$ at $x=5$ is given by

$$f^{(n)}(5)=\frac{(-1)^n n!}{2^n(n+2)} \text{ and } f(5)=\frac{1}{2}$$

Show that the sixth-degree Taylor polynomial for $f$ about $x=5$ approximates $f(6)$ with an error less than $\frac{1}{1000}$.

15. The population $P(t)$ of unicorns in a forest satisfies the logistic differential equation $\frac{dP}{dt}=3P-\frac{P^2}{6000}$.

(a) If $P(0)=4000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(b) If $P(0)=10,000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(c) If $P(0)=20,000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(d) If $P(0)=4000$, what is the population when it is growing the fastest? Where is the solution curve concave up? Concave down? Justify your answer.

16. (Calculator Permitted) Given $\frac{dx}{dt}=\cos\left(t^3\right)$ and $\frac{dy}{dt}=3\sin\left(t^2\right)$ for $0\le t\le 3$. At time $t=2$, the object at position $(4,5)$.

(a) Find the speed of the object at time $t=2$.

(b) Find the total distance traveled by the object over the time interval $0\le t\le 1$.

(c) Find the position of the object at time $t=3$.

17. (Calculator Permitted) Given $f(x)=\frac{1}{3}+\frac{2x}{9}+\frac{3x^2}{27}+\cdots+\frac{(n+1)}{3^{n+1}}x^n+\cdots$

(a) $\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x}$.

(b) Write the first three nonzero terms and the general term for the infinite series that represents $\int_0^1 f(x)dx$.

(c) Find the sum of the series found in part (b).

18. (Calculator Permitted) The function $f$ has derivatives of all orders for all real numbers $x$. Assume that $f(2)=5$, $f'(2)=-3$, $f''(2)=4$, $f'''(2)=-1$, and $\left|f^{(4)}(x)\right|\le 3$ for all $x$ in $[2,2.2]$.

(a) Write the third-degree Taylor polynomial for $f$ about $x=2$.

(b) Use your answer to (a) to approximate $f(2.15)$. Give your answer correct to five decimal places.

(c) Use the Lagrange error bound on the approximation of $f(2.15)$ to explain why $f(2.15)\ne 4.7$.