1. (2011-BC3) Let \( f(x) = e^{2x} \). Let \( R \) be the region in the first quadrant bounded by the graph of \( f \), the coordinate axes, and the vertical line \( x = k \), where \( k > 0 \). The region \( R \) is shown in the figure above.
   (a) Write, but do not evaluate, and expression involving an integral that gives the perimeter of \( R \) in terms of \( k \).
   (b) The region \( R \) is rotated about the \( x \)-axis to form a solid. Find the volume, \( V \), of the solid in terms of \( k \).
   (c) The volume \( V \), found in part (b), changes as \( k \) changes. If \( \frac{dk}{dt} = \frac{1}{3} \), determine \( \frac{dV}{dt} \) when \( k = \frac{1}{2} \).

2. (2011-BC6) Let \( f(x) = \sin(x^2) + \cos x \). The graph of \( y = \left| f^{(5)}(x) \right| \) is shown above.
   (a) Write the first four nonzero terms of the Taylor series for \( \sin x \) about \( x = 0 \), and write the first four nonzero terms of the Taylor series for \( \sin(x^2) \) about \( x = 0 \).
   (b) Write the first four nonzero terms of the Taylor series for \( \cos x \) about \( x = 0 \). Use this series and the series for \( \sin(x^2) \), found in part (a), to write the first four nonzero terms of the Taylor series for \( f \) about \( x = 0 \).
   (c) Find the value of \( f^{(6)}(0) \).
   (d) Let \( P_4(x) \) be the fourth-degree Taylor polynomial for \( f \) about \( x = 0 \). Using information from the graph of \( y = \left| f^{(5)}(x) \right| \) shown above, show that \( \left| P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right) \right| < \frac{1}{3000} \).
3. (2011B-BC2) (Calculator Permitted) The polar curve \( r \) is given by \( r(\theta) = 3\theta + \sin \theta \), where \( 0 \leq \theta \leq 2\pi \).
(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of \( r \).
(b) For \( \frac{\pi}{2} \leq \theta \leq \pi \), there is one point \( P \) on the polar curve \( r \) with \( x \)-coordinate \(-3\). Find the angle \( \theta \) that corresponds to point \( P \). Find the \( y \)-coordinate of point \( P \). Show the work that leads to your answers.
(c) A particle is traveling along the polar curve \( r \) so that its position at time \( t \) is \((x(t), y(t))\) and such that \( \frac{d\theta}{dt} = 2 \). Find \( \frac{dy}{dt} \) at the instant that \( \theta = \frac{2\pi}{3} \), and interpret the meaning of your answer in the context of the problem.

\[
f(x) = \begin{cases} \cos x - \frac{1}{2} & \text{for } x \neq 0 \\ x^2 & \text{for } x = 0 \end{cases}
\]

4. (2010-BC6) The function \( f \), defined above, has derivatives of all orders. Let \( g \) be the function defined by \( g(x) = 1 + \int_0^x f(t) \, dt \).
(a) Write the first three nonzero terms and the general term of the Taylor series for \( \cos x \) about \( x = 0 \). Use this series to write the first three nonzero terms and the general term of the Taylor series for \( f \) about \( x = 0 \).
(b) Use the Taylor series for \( f \) about \( x = 0 \) found in part (a) to determine whether \( f \) has a relative maximum, relative minimum, or neither at \( x = 0 \). Give a reason for your answer.
(c) Write the fifth-degree Taylor polynomial for \( g \) about \( x = 0 \).
(d) The Taylor series for \( g \) about \( x = 0 \), evaluated at \( x = 1 \), is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for \( g \) about \( x = 0 \) to estimate the value of \( g(1) \). Explain why this estimate differs from the actual value of \( g(1) \) by less than \( \frac{1}{6!} \).

5. (2010B-BC5) Let \( f \) and \( g \) be the functions defined by \( f(x) = \frac{1}{x} \) and \( g(x) = \frac{4x}{1 + 4x^2} \), for all \( x > 0 \).
(a) Find the absolute maximum value of \( g \) on the open interval \((0, \infty)\) if the maximum exists. Find the absolute minimum value of \( g \) on the open interval \((0, \infty)\) if the minimum exists. Justify your answers.
(b) Find the area of the unbounded region in the first quadrant to the right of the vertical line \( x = 1 \), below the graph of \( f \), and above the graph of \( g \).

6. (2010B-BC6) The Maclaurin series for the function \( f \) is given by \( f(x) = \sum_{n=2}^{\infty} \frac{(-1)^n (2x)^n}{n-1} \) on its interval of convergence.
(a) Find the interval of convergence for the Maclaurin series of \( f \). Justify your answer.
(b) Show that \( y = f(x) \) is a solution to the differential equation \( xy' - y = \frac{4x^2}{1 + 2x} \) for \( |x| < R \), where \( R \) is the radius of convergence from part (a).