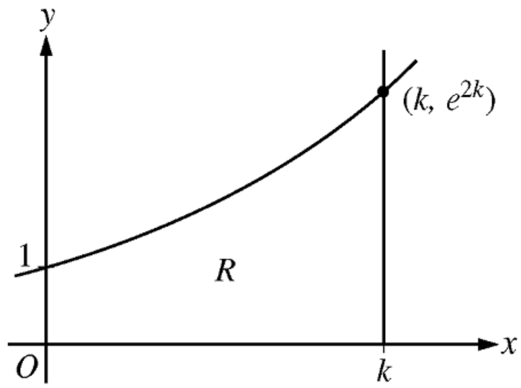
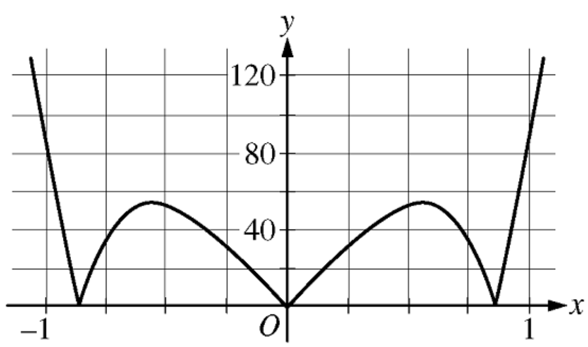
BC Review Pre-FINAL Review, NO Calculator unless otherwise noted.
Do all work on separate notebook paper



1. (2011-BC3) Let $f(x)=e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x=k$, where $k>0$. The region R is shown in the figure above.
   (a) Write, but do not evaluate, and expression involving an integral that gives the perimeter of R in terms of k.
   (b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.
   (c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt}=\frac{1}{3}$, determine $\frac{dV}{dt}$ when $k=\frac{1}{2}$.



Graph of $y=\left|f^{(5)}(x)\right|$

2. (2011-BC6) Let $f(x)=\sin\left(x^2\right)+\cos x$. The graph of $y=\left|f^{(5)}(x)\right|$ is shown above.
   (a) Write the first four nonzero terms of the Taylor series for $\sin x$ about $x=0$, and write the first four nonzero terms of the Taylor series for $\sin\left(x^2\right)$ about $x=0$.
   (b) Write the first four nonzero terms of the Taylor series for $\cos x$ about $x=0$. Use this series and the series for $\sin\left(x^2\right)$, found in part (a), to write the first four nonzero terms of the Taylor series for f about $x=0$.
   (c) Find the value of $f^{(6)}(0)$.
   (d) Let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x=0$. Using information from the graph of $y=\left|f^{(5)}(x)\right|$ shown above, show that $\left|P_4\left(\frac{1}{4}\right)-f\left(\frac{1}{4}\right)\right|<\frac{1}{3000}$

3. (2011B-BC2) (Calculator Permitted) The polar curve r is given by $r(\theta)=3\theta+\sin\theta$, where $0\le\theta\le 2\pi$.
   (a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.
   (b) For $\frac{\pi}{2}\le\theta\le\pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.
   (c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt}=2$. Find $\frac{dy}{dt}$ at the instant that $\theta=\frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

$$f(x)=\begin{cases}\dfrac{\cos x-1}{x^2} & \text{for } x\ne 0\\ -\dfrac{1}{2} & \text{for } x=0\end{cases}$$

4. (2010-BC6) The function f, defined above, has derivatives of all orders. Let g be the function defined by $g(x)=1+\int_0^x f(t)\,dt$.
   (a) Write the first three nonzero terms and the general term of the Taylor series for $\cos x$ about $x=0$. Use this series to write the first three nonzero terms and the general term of the Taylor series for f about $x=0$.
   (b) Use the Taylor series for f about $x=0$ found in part (a) to determine whether f has a relative maximum, relative minimum, or neither at $x=0$. Give a reason for your answer.
   (c) Write the fifth-degree Taylor polynomial for g about $x=0$.
   (d) The Taylor series for g about $x=0$, evaluated at $x=1$, is an alternating series with individual terms that decrease in absolute value to 0. Use the third-degree Taylor polynomial for g about $x=0$ to estimate the value of $g(1)$. Explain why this estimate differs from the actual value of $g(1)$ by less than $\frac{1}{6!}$.

5. (2010B-BC5) Let f and g be the functions defined by $f(x)=\frac{1}{x}$ and $g(x)=\frac{4x}{1+4x^2}$, for all $x>0$.
   (a) Find the absolute maximum value of g on the open interval $(0,\infty)$ if the maximum exists. Find the absolute minimum value of g on the open interval $(0,\infty)$ if the minimum exists. Justify your answers.
   (b) Find the area of the unbounded region in the first quadrant to the right of the vertical line $x=1$, below the graph of f, and above the graph of g.

6. (2010B-BC6) The Maclaurin series for the function f is given by $f(x)=\sum_{n=2}^{\infty}\frac{(-1)^n(2x)^n}{n-1}$ on its interval of convergence.
   (a) Find the interval of convergence for the Maclaurin series of f. Justify your answer.
   (b) Show that $y=f(x)$ is a solution to the differential equation $xy'-y=\frac{4x^2}{1+2x}$ for $|x|<R$, where R is the radius of convergence from part (a).