Chapter P: What is AP Calculus?

A GRAFFITIED BOVINE?


A PAINFUL STONE?

Congratulations on your decision to take AP Calculus. You are now in the elite group (one of approximately 250,000) of high school students who will be spending many hours each and every night working homework problems, finding limits, differentiating complex equations, and integrating numerically from a set of day while others are watching TV, hanging out with friends, and doing other futile things to try to fill the mathematical void they feel. During the next nine months, you will be investing a great amount of time, energy, passion, sweat, and perhaps even a little blood (paper cuts?) learning profound mathematical concepts and skills, topics that your mathematical forefathers would have given their lives to know.

I know AP Calculus can seem like such a daunting discipline, but relax. Calculus is ***not*** a four-letter word. In fact, it's an eight-letter word—that's twice as bad, right?! Relax, "painless" and "pleasant" are also eight-letter words. So before we start this painlessly pleasant journey into AP Calculus, here are the answers to some of the questions you might have to put you further at ease.

Why on Earth am I in this class?

Hopefully you're sitting in this class for the right reason: either your parents are forcing you to take it because they had to suffer through it when they were your age, your counselor stuck you in this class on accident thinking it was "Computers," or you just signed up to be with all your "mathy" friends. Ideally you are here on your own accord. Whether it's because you want to actually learn calculus and prepare for life at the next academic level, you heard the class was an easy "A," or you're just doing it to get the weight points for class ranking purposes, understand one very important thing—those who can do ***math*** can do ***anything***. In fact, widespread research confirms that those students who simply take AP Calculus do significantly better in college, with higher graduation rates, graduating in fewer years, earning better grades, landing better jobs after college, and living overall more-productive, more-satisfying, and extraordinary lives. The research indicates even ***better*** results for those who actually take and pass the AP exam.

Do I have to take the AP Calculus exam?
If you stay in this AP Calculus class the entire year, then this question has the same answer as the following questions: Do I ever have to get my driver's license? If I tear all the ligaments in my knee, will I need surgery? Will I fail this course if I don't do any homework? Taking the exam is not a requirement, but it is an expectation.

Is the AP exam hard?
Absolutely not! Diamonds are hard. The AP exam is much softer. It *is* challenging, though, especially if you take it without the expectation of earning a five for yourself. Everything in life is difficult for those who are unwilling. Begin your study of AP Calculus with the genuine expectation that you can and *will* earn the top score. I can *promise* that if you work hard in this course and give it the respect it deserves, you will be equipped with everything you will need to earn a FIVE!

Can I cram for a calculus test?
Can you cram for a basketball free-throw shooting contest? Can you cram for a piano recital? Can you cram for a chess tournament? Truthfully, you can cram for anything, but you may not get the results you hope for.

Since mathematics is a combination of concepts and skills, not just names and dates, the best way to prepare for any math exam is by first memorizing any important terms, formulas, notations, and reading through any theorems ***before*** beginning any practice exercises. The recipe for success is then the same as the recipe for an aspiring pianist to reach Carnegie Hall: practice, practice, practice . . . (then turn right). Imagine trying to play competitive chess when you've never played or even know the rules. You're not going to be effective, much less efficient, if you play with the "horse piece" in one hand and a "Chess for Dummies" book in the other. You're going to lose and frustrate your playing partner. Working with such myopic vision from one move to the next, you'll never see the big picture, much less develop your own strategies based on your experience and insight into the intricate nuances of the game. You're just a monkey shuffling pieces on a Cartesian plane.

To overstate another analogy, think of playing football. You first learn the plays and formations. Then you practice those plays. Practicing the plays is analogous to doing your math homework after learning the definitions and theorems. Depending on your own skill level and the sophistication of the play itself, sometimes you'll practice harder and longer than you previously did, sometimes longer than others do. As the big football game approaches, you might have a scrimmage to simulate game play. These mirror the in-class quizzes which help you to identify where you need to focus your final efforts. The day before the big game is usually just a half-pad practice, a final run-through just to stay sharp. The "half-pad practice" the night before any math exam should be spent going over your old practice problems, rereading your class notes, and mentally quizzing yourself on theorems, formulas, etc. When the day of the big game finally arrives, you are confident and filled with the exuberant energy of anticipation. You go out focused and determined. You play hard and well. Your moves and reactions are almost subconscious and instinctual at this point. Your patient pertinacity and diligence in practicing pays off. The same can be said on the day of the big math exam: you simply show up to class and celebrate the knowledge and skills you've dedicated yourself to mastering. You have fun. And just like a big football game, on a math test, there's no chance to play the game again. No do-overs. No retests. It all takes place in that tiny crucible of a moment. As Jaime Escalante said, when the time comes, you must "Stand and Deliver" . . . or in the case of math test, "sit" and deliver.

What's the difference between AP Calculus AB and AP Calculus BC?

They're both very similar in that they both involve the study of calculus, both require a great deal of hard work, and both have a "B" in their title. The real *difference* is that AB typically covers the content covered in a first semester college course while BC covers the first two semesters of a college course. The depth at which topics are covered is equivalent in both courses, but BC, having to cover almost twice the material, moves at almost twice the velocity. Students may take BC after successfully completing an AB course or, for those brave few, jump right into it out of PreAP Precalculus. The AP BC exam is 60% AB topics and 40% BC-only topics. Students taking the BC exam will receive an AB subscore, meaning they can earn 3 hours of college credit for mastering the AB topics without mastering the BC topics. A passing score on the entire BC exam means the student has mastered both portions and can receive up to 6 hours of college credit, depending on the particular college and individual degree requirements.

Will I have to do any writing in AP Calculus?

Of course! You'll be writing many, many numbers, but you'll also be writing many words around those numbers. The AP exam consists of a multiple choice section, where no credit is given for work, and where a thoughtfully arrived at correct answer is indistinguishable from a "lucky guess." The second section is a free-response section, where "lucky guesses" are less likely. The free-response section is your opportunity to clearly ***communicate*** correct mathematical procedures and calculations. You will be asked to "justify your answer" or "explain your reasoning." While you won't be asked to write a "compare and contrast" essay, you will have to write in complete sentences, usually containing an introductory adverbial clause and clearly defined antecedents to your prepositions (in fact, we'll eschew prepositions altogether . . . you'll see.) To train for this, you will be required to summarize and defend your results throughout the year using proper grammar, spelling, and punctuation. The philosophy is that if you get in the habit of dotting your "i"s and crossing your "t"s when you write, you'll be less likely to miss distributing a negative sign or making any number of egregious algebraic faux pas when you perform mathematical calculations. Think of it as doing "Hi-Def" mathematics—it's all about careful attention to detail.

Will I need a fancy calculator?

You will not need anything as fancy as one that's powered entirely by the sun. You will be able to get through the entire course with a calculator from the TI-83/84 family. These calculators are super-powered number crunchers that will handle every computation the AP Calculus exams will require you to do. Calculators such as the TI-89, equipped with a computer algebraic system (CAS) are legal, more expensive, but almost too powerful. With power comes great responsibility, and if you have a TI-89, your responsibility is to not rely on its power too much. While the 89s do symbolic math, like factoring a cubic polynomial for you, the AP Calculus exam is carefully written so that students using an 89 over an 84 or 83 have no computational advantage. In any case, you do not want to develop a calculator dependency. Too many students use the calculator as an unnecessary crutch, others as a stretcher. In this course, you will learn how to use your calculator as a tool.

Can I store programs on my calculator?

Your calculator's memory will ***not*** be cleared on the day of the AP exam, so any program in your calculator's memory may be accessed on the two calculator portions of the exam. However, the clever test writers allow this only because the questions on which calculators are permitted are written in such a way that programs are of no use. Accessing formulas on a calculator also squanders valuable time that could be used completing or checking other parts of the exam. It is best that you memorize everything you need to then practice those things repeatedly. When you master them in a problem-based context, you will eventually retrieve them much faster than you could retrieve a calculator program. Storing formulas on your calculator is not illegal, but if you've been taking care of business in the classroom on a regular basis, you will know these formulas like the back of your hand (if the back of the hand is something that you know very well) thwarting any last-minute brain flatulence come test time. In reality, it's less likely that your brain's batteries will die before your calculator's will.

Should I round to two decimals?

Absolutely not! For approximate answers, the AP exam requires at least a ***3 decimal accuracy*** (rounded or truncated). You may leave answers exact (simplified or unsimplified) or you may report more than 3 decimals (as long as they are the correct decimal digits). At no point, though, should you ever use a rounded number to calculate another number. In the bridge building profession, they call that "total failure" or "major lawsuit."

When will I ever use Calculus?

NEVER . . . if you don't master it. Aside from needing calculus to do well on tests so that you can earn fantastic grades to get into the college of your choice, you might need calculus either to satisfy your college degree requirement and/or in your chosen profession (i.e. engineer, mathematician, high school math teacher, or consultant on a hit prime-time TV show about math). Even if you never again use calculus, per se, know that it is all around you, and the disciplined habits of mind that you forge in mastering the concepts and skills of calculus transcend any classroom and profession, freeing your mind from naiveté and prejudices. Take solace in knowing that you are now a poet of logical ideas and master of the language in which the entire universe is written. Remember . . . if you can do math, you can do anything!

What in the world is Calculus anyway? Did it fall out of the sky?

Calculus is the study of change and variability, and yes, it DID fall out of the sky . . . sort of. In the 17th century, scientists were trying to work out a way to predict how planets move through their orbits. The paths were known to be elliptical, but because planets traveled through their orbits at variable speeds, a method for systematically predicting the planet's behavior was elusive. That is until Edmond Halley, the comet guy, met with Isaac Newton in the summer of 1684 and expressed his frustrations. As if it were nothing, Newton retrieved a piece of paper with some calculations he had done prior to the visit. It was exactly what the astronomers needed. It was a new math, which Newton called "**fluxions**" that later became the Calculus we will study. Imagine to have discovered such an amazing thing as calculus only to sit on it! But this was Newton. I guess when every idea you have is brilliant, it becomes difficult to think anything of them. This year, hopefully, you'll be filled with many brilliant ideas of your own as you study the calculus that Newton discovered. We will use the calculus to answer two basic questions: How can we find instantaneous rates of change? and How can we find areas of irregular regions? These two seemingly unrelated questions have one very important thing in common involving something infinitely small but hugely important, as you will soon see.

So Newton invented Calculus?

Not exactly. He co-*discovered* it. *Machines* are invented. *Mathematics* is unveiled. In the mid 1670s, a German philosopher, mathematician, and fashion guru named Gottfried Leibniz discovered another version of calculus. Publishing his findings in 1684, the same year Halley met with Newton, Leibniz's method was a bit different than Newton's "fluxions." Leiniz's calculus was called the

method of infinitesimals, used completely different notation than Newton, and came about from the necessity to answer questions about finding slopes of tangent lines on graphs—not planetary motion.

Newton's official publication of his "fluxions," called the ***Principia***, was not until 1687, roughly three years after Leibniz went to press. Both Newton and Leibniz claimed to be the original discoverers. Both claimed the other stole his ideas. Both had reputable supporters. Believe it or not, the priority dispute still exists today, but the majority of people who even care (which is a very small minority) are happy to give each man his own due credit and write off the dispute to the coincidental fact that one of the greatest discoveries of mankind came about independently at the same time by two men in two different parts of the world who were motivated by two different things.

Poor Leibniz, though, never got his due credit in his lifetime, as Newton went on to become head of the English Royal Society, the gatekeepers of all intellectual pursuits of the day, and he made sure Leibniz became less than a footnote in the annals of history. The irony, though, is that the calculus we study today, officially called Newtonian Calculus, does not use Newton's own notation, but rather the cleaner, more beautiful notation of Leibniz. If Leibniz were alive today, he'd be very, very old and laughing satisfactorily.

Does AP Calculus have its own conventions, jargon, and lexicon?
You betcha! With anything technical comes new methods and terminology, but it also helps to ensure consistency, accuracy, and fairness to all students who end up taking the AP Calculus exam. Here are a few.

Copy Error: Miscopying the stated problem, then working it correctly –or- miscopying your work from one line to the next. If all else is done correctly and the error doesn't result in a radically simpler or different type of problem, a one point deduction is usually incurred.

Saying too Much: Solving a given problem correctly and answering the stated question, but then doing additional work or making additional, unsolicited statements, which may be incorrect. If the extra information is correct, no harm, no foul. However, if the additional work is incorrect, a deduction will be taken.

Parallel Solutions: Presenting two or more solutions to a problem without choosing which one should be graded. In this case, each solution is graded independently and the average of the two scores becomes the final score. In this scenario, the score is always rounded DOWN!

Crossed-out Work: This type of work is "invisible" to anyone who will be grading your work. Do not cross out any work unless you have intentions and time to present another solution!!!!

Three-decimal-place rule: Unless otherwise specified, students, when reporting decimal answers, must have accuracy to the third decimal place (thousandths) either rounded or truncated. Decimal accuracy beyond three places is not read.

No Simplification Needed: An answer (numeric OR algebraic) need not be simplified to be given full credit. For example, if a correct solution to a problem results in $e^0 - 4 + 6 + \tan \pi$, and the student leaves the answer in that form, the student earns full credit.

Immunity from further deductions: In a single problem, errors in decimal presentation or in units, may receive only a one-point deduction for that problem. This decision is made from year to year and may change from one problem/year to the next. For instance, in one problem, the units might be considered part of the correct answer. Each missing unit, therefore, results in a point deduction. Sometimes, however, the units themselves are worth one point. Miss them as many times as you want for the same low price of one point deduction.

No recipes or formulas: Merely writing down a theorem or a formula receives no credit. Students must apply the procedure described tailored toward the specific problem at hand.

No calculator syntax: On a calculator-active question, when a student presents a solution to an equation, a value of a derivative, or a value of a definite integral, the setup of that equation, derivative, or integral must be presented in valid mathematical notation, NOT just calculator syntax. Examples:

OK	OK	NOT OK
$\int_2^5 (3x+1)\,dx$	$fn\text{int}(3x+1, x, 2, 5)$ $\int_2^5 (3x+1)\,dx$	$fn\text{int}(3x+1, x, 2, 5)$

Arithmetic and algebra errors: non-calculus errors

Bald Answers: Any answer without supporting work is considered bald. In most cases, bald answers are not given any credit, and definitely not accepted when supporting work is expected.

Eligibility Requirements: Students can receive partial credit for work done later in a problem even if they've made a mistake in an earlier part of the problem. Usually an intermediate step or process must be evident to be eligible for partial credit.

Our problem: The problem stated on the exam. If students are not working our problem, no points are awarded. For instance, if the problem says to solve a definite integral and the study solves the problem of world hunger, no points are awarded. Similarly, if a copy error changes a problem with a logarithm into a simpler linear polynomial problem, the student becomes ineligible for any points.

Read with the student: In general, if a student makes an error that does not significantly alter the intent of the problem, students can receive partial credit from that point forward. The AP exam does not like to "double whammy" a student who makes a single mistake while clearly demonstrating knowledge of the concept. Never give up on a problem.

Recoup: A student loses a point, say for a copy error, but corrects the copy error at some legitimate and reasonable point later in the problem. That missed point may be recouped.

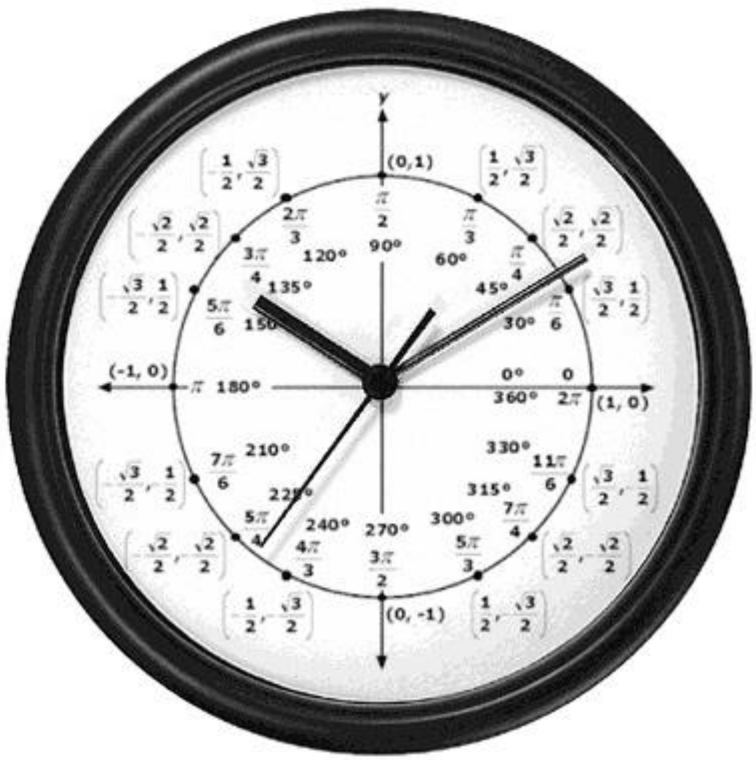
Restart: A student begins a problem (but doesn't finish it), abandons this initial attempt, then starts the problem again often with a different or unrelated approach. Even if the initial attempt is not crossed out, full credit can be obtained. Note how this is different from parallel solutions.

Reversal: When two terms are transposed, such as writing $a-b$ rather than $b-a$. A point is lost, but students are usually eligible for full credit beyond that point as the grader reads with the student (see above.)

Misuse of equality: When two expressions that aren't equal to each other are set equal to each other. This usually happens out of carelessness and haste rather than from the actual belief that they are equal. This ALWAYS results in a point deduction.

So who invented Precalculus?

That's a funny question. Of course it was invented simultaneously by Newton's and Leibniz's fathers (insert laugh track here). Seriously though, precalculus is not a branch of mathematics but rather the name of the course that typically precedes one's study of calculus. It consists of function analysis and trigonometry. With a name like "Precalculus," you can probably guess that the concepts, information, skills, and algebraic techniques you mastered in precal are going to be the essential machinery that you'll need to master calculus. This includes remembering the Unit Circle. The next chapter will review you on some of the essentials you'll want to review (in case you're got a little rusty over summer).



It's time to BOTC
(Bust Out The Calculus)