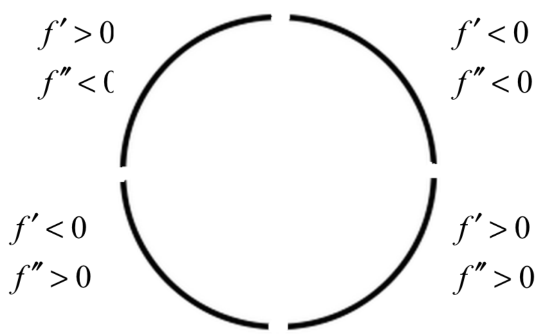
§3.5—f, f', f''

One of the great appliations of the calculus we've learned so far is how to use the relation among f, f', f'' sketch a graph of a function $f(x)$.

		0	Positive	Negative
f	y-values of the function	root/zero of f /x-intercept	graph of f is above the x-axis	graph of f is below the x-axis
f'	how the y-values are changing/slopes	critical values of f /possible locations of relative max/mins	graph of f is increasing	graph of f is decreasing
f''	how the slopes are changing/concavity	possible point of inflection/possible location of inflection points	graph of f is concave up/slopes of f are increasing	graph of f is concave down/slopes of f are decreasing

Recall from the previous section. If we know the sign of f' and f'' on a given interval, we know exactly what the graph of f looks like on that interval. If we then know the y-values of f, we know exactly where on the coordinate grid to graph it.



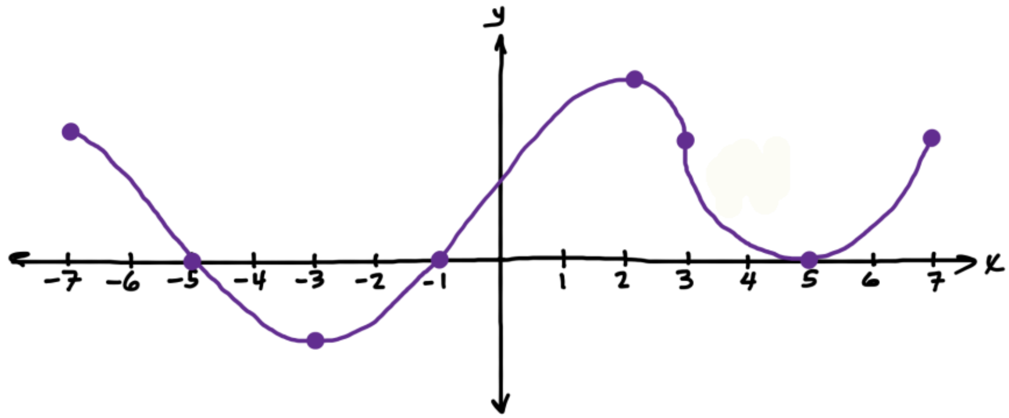
Example 1:
Sketch a possible graph of a function f that satisfies the following conditions:

(i) $f'(x) > 0$ on $(-\infty, 1)$, $f'(x) < 0$ on $(1, \infty)$

(ii) $f''(x) > 0$ on $(-\infty, -2) \cup (2, \infty)$ and $f''(x) < 0$ on $(-2, 2)$.

(iii) $\lim_{x \to -\infty} f(x) = -2$ and $\lim_{x \to \infty} f(x) = 0$

Example 2:



(a) If the graph above is the graph of a function $f(x)$ on $[-7,7]$,

(i) On what open interval(s) is $f(x)$ both decreasing and concave down?

(ii) On what open interval(s) is $f(x)$ both increasing and concave down?

(iii) At what x-value(s) does $f(x)$ have inflection points?

(b) If the graph above is the graph of a function $f'(x)$ on $[-7,7]$,

(i) On what open interval(s) is $f(x)$ increasing?

(ii) At what x-value(s) does $f(x)$ have a local minimum? Justify.

(iii) On what open interval(s) is $f(x)$ concave up?

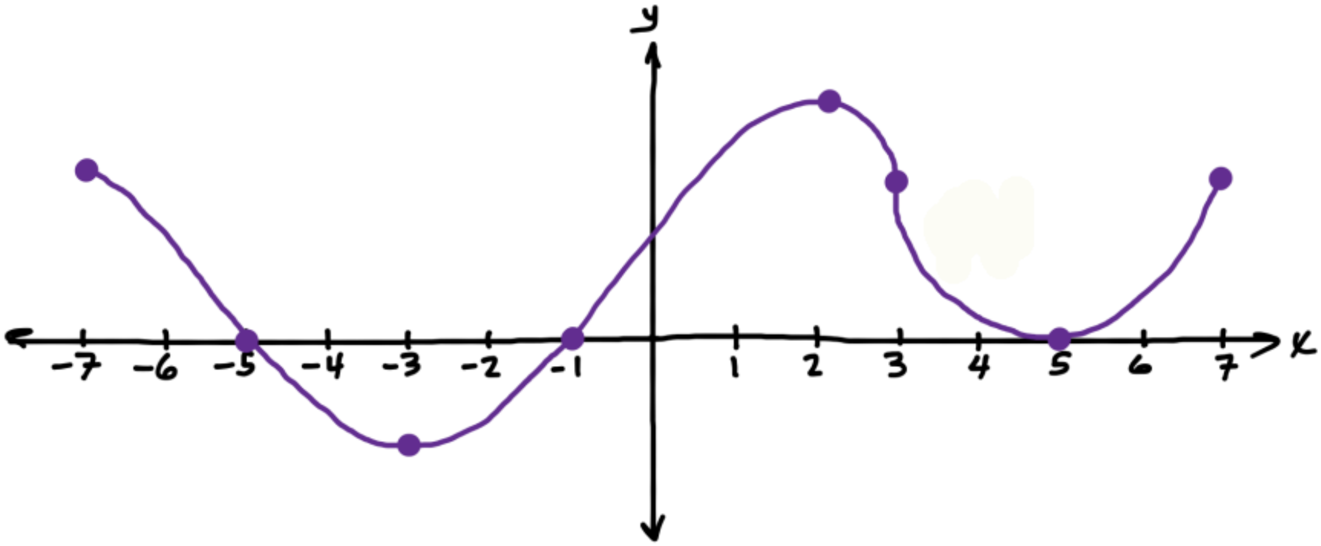
(iv) At what x-value(s) does $f(x)$ have an inflection point? Justify.

(c) If the graph above is the graph of a function $f''(x)$ on $[-7,7]$,

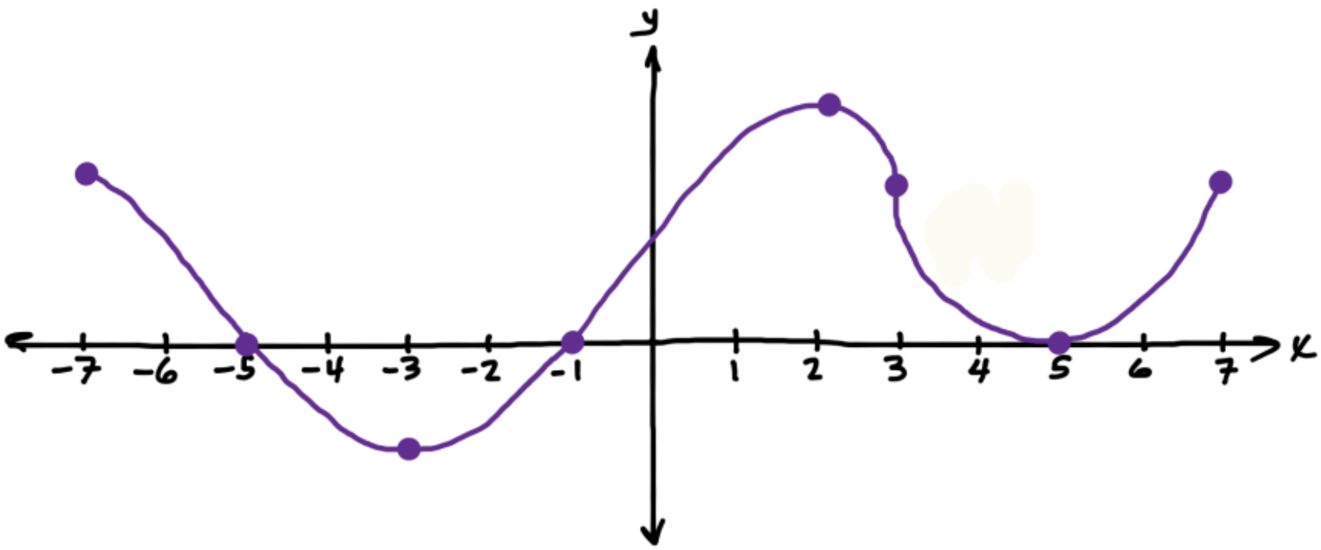
(i) On what open interval(s) is $f(x)$ both concave down?

(ii) At what x-value(s) does $f(x)$ have an inflection point? Justify.

Example 3:



(a) The graph above is the graph of a function $f(x)$ on $[-7,7]$. On the same set of axes, sketch and label a possible graph of f' and f''.

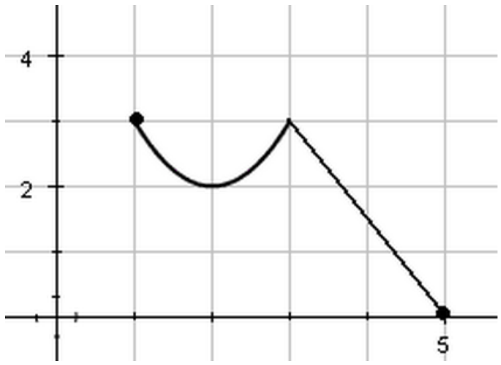


(b) The graph above is the graph of a function $f'(x)$ on $[-7,7]$. On the same set of axes, sketch and label a possible graph of $f(x)$ given that $f(-7)=0$.

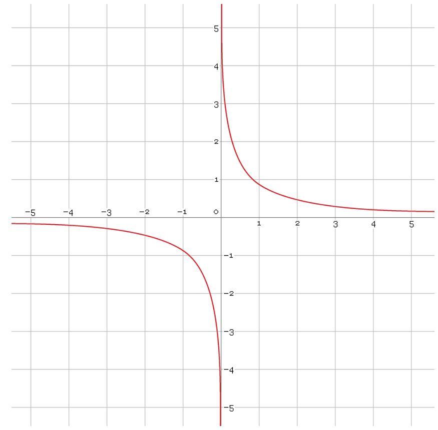
Example 4:

For each of the following graphs of $f'(x)$, sketch a possible graph of both $f(x)$ and $f'(x)$.

(a)



(b)



Example 5:

(AB2 1982) Given that f is the function defined by $f(x)=\dfrac{x^3-x}{x^3-4x}$

(a) Find $\lim\limits_{x\to 0} f(x)$.

(b) Find the zeros of f .

(c) Write an equation for each vertical and each horizontal asymptote to the graph of f .

(d) Describe the symmetry of the graph of f . Justify algebraically.

(e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f .

Example 6:

Use information provided by f , f' , f'' to sketch a graph of the function $f(x)$ for each of the following.

(a) $f(x) = -x^5 + \frac{5}{2}x^4 + \frac{40}{3}x^3 + 5$

(b) $f(x) = 2x^{5/3} - 5x^{4/3}$

(c) $f(x) = \frac{x^2 - 2x + 4}{x - 2}$

(d) $f(x) = \dfrac{x}{\sqrt{x^2+2}}$

.

(e) $f(x) = e^{4-x^2}$