§4.1—Antiderivatives & Indefinite Integration

Suppose we have a function *F* whose derivative is given as $F'(x) = f(x) = x^2$. From your experience with finding derivatives, you might say that $F(x) =$ WHAT???? How can you check your answer?????

Congratulations, you have just found *an* **antiderivative**, *F*, of *f*.

Definition

A function F is *an* **antiderivative** of *f* on an interval *I* if $F'(x) = f(x) \quad \forall x \in I$.

Notice that *F* is called AN antiderivative and not THE antiderivative. This is easily understood by looking at the example above.

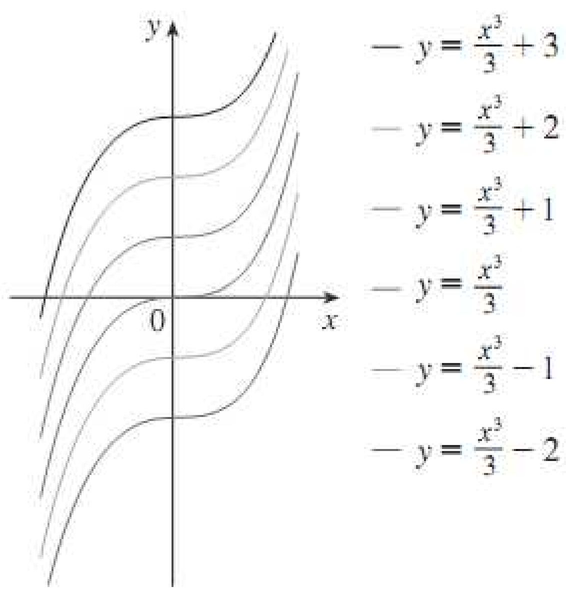
Some antiderivatives of $f(x) = x^2$ are $F(x) = \frac{1}{3}x^3$, $F(x) = \frac{1}{3}x^3 + 3$, $F(x) = \frac{1}{3}x^3 - 2$, and $F(x) = \frac{1}{3}x^3 + ®$ because in each case, $\frac{d}{dx}\left[F(x)\right] = x^2$.

Because of this we can say that the **<u>general antiderivative</u>** of a function $f(x)$ is

$F(x) + C$, where *C* is an arbitrary constant.

The graph at right show several members of the family of the antiderivatives of x^2.

WHAT GRAPHICAL CONSEQUENCE DOES THE +*C* HAVE ON THE SOLUTION CURVES?



Example 1:
Find the general antiderivatives of each of the following using you knowledge of how to find derivatives.

(a) $f(x) = 2x$ (b) $f'(x) = x$ (c) $F'(x) = \frac{2}{3}x^{-\frac{4}{7}}$ (d) $g'(x) = \frac{1}{x^2}$ (e) $\frac{dy}{dx} = \cos x$

Knowing how to find a derivative of different types of functions will help you find antiderivatives.

Table of Antiderivative Formulas

Function	General antiderivative		Function	General antiderivative
$cf(x)$	$cF(x)+C$		$\csc^2 x$	$-\cot x+C$
$f(x)\pm g(x)$	$F(x)\pm G(x)+C$		$\sec x\tan x$	$\sec x+C$
$x^n,\ n\neq -1$	$\frac{1}{n+1}x^{n+1}+C$		$\csc x\cot x.$	$-\csc x+C$
$\frac{1}{x}$	$\ln\lvert x\rvert+C$		$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x+C$
e^x	e^x+C		$\frac{1}{1+x^2}$	$\tan^{-1} x+C$
$\cos x$	$\sin x+C$		$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1}\lvert x\rvert+C$
$\sin x$	$-\cos x+C$			
$\sec^2 x$	$\tan x+C$			

Example 2:

Find all functions g such that $g'(x)=4\sin x+\frac{2x^4-\sqrt{x}+x}{x}-\frac{7x\csc^2 x+1}{x}$.

Definition

A **differential equation** is an equation that has a derivative in it. **Solving a differential equation** involves finding the original function from which the derivative came. The **general solution** involves $+C$. The **particular solution** uses an initial condition to find the specific value of C.

Example 3:
Solve the differential equation $f'(x) = 3x^2 + 1$ if $f(2) = -3$. Find both the general and particular solutions.

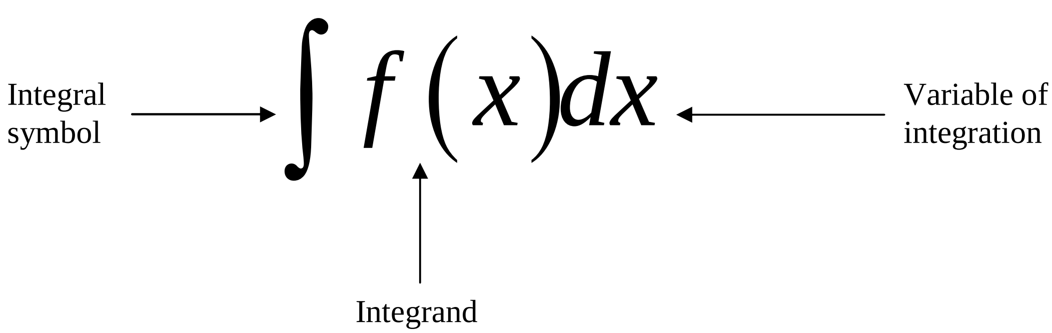
Example 4:
Find the particular solution to the following differential equation if $\frac{dy}{dx} = e^x + 20\left(1+x^2\right)^{-1}$ and $y(0) = -2$.

When we are asked to take the derivative of an expression, we have the verb notation

$$\frac{d}{dx}\left[f(x)\right] =$$

We now need an equivalent verb expression that indicates that we find the antiderivative. It is called the indefinite integral.

Here's the anatomy of an indefinite integral:



*Because the indefinite integral gives the antiderivative, integration and antidifferentiation are mathematical synonyms, and an indefinite integral is equivalent to a general antiderivative.

Example 5:

Find the particular solution to the following differential equation if $\dfrac{d^2y}{dx^2} = 12x^2 + 6x - 4$ and

(a) $y'(1) = 3$ and $y(0) = -6$

(b) $y(0) = 4$ and $y(1) = 1$.

Sometimes we need to manipulate our integrand into something more recognizable.

Example 6:

Evaluate each of the following:

(a) $\displaystyle\int \left[\frac{5\sqrt{1-x^2}}{3-3x^2} \right] dx$

(b) $\displaystyle\int \frac{\sin t}{\cos^2 t} dt$

(c) $\displaystyle\int \left(\tan^2 p + 4 \right) dp$

(d) $\displaystyle\int 3\cos^2\left(\frac{m}{2}\right) dm$

(e) $\displaystyle\int z^3 (3-2z)^2 \, dz$

(f) $\displaystyle\int \left[\frac{x^2 - 5x - 14}{x-7} \right] dx$