

§7.2—Parametric & Vector Accumulation

Displacement & Distance Traveled

Suppose a particle moves along a path in the plane so that its velocity at any time t is $\vec{v}(t) = (x'(t), y'(t))$, then the **displacement** from $t = a$ to $t = b$ is given by the vector

$$\left\langle \int_a^b x'(t) dt, \int_a^b y'(t) dt \right\rangle.$$

The preceding vector is added to the position at time $t = a$ to get the **position** at time $t = b$.

The **distance traveled** from $t = a$ to $t = b$ is the arc length

$$\int_a^b |\vec{v}(t)| dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2} dt.$$

Example 1:

(Calculator) An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = t \sin(t), \quad \frac{dy}{dt} = \cos(t^2). \quad \text{At time } t = 2, \text{ the object is at the position } (1, 4).$$

- Find the acceleration vector for the object at $t = 2$.
- Write the equation of the tangent line to the curve at the point where $t = 2$.
- Find the speed of the object at $t = 2$.

d) Find the displacement of the object from $t = 2$ to $t = 7$.

e) Find the distance traveled by the object from $t = 2$ to $t = 7$.

f) Find the position of the object at time $t = 1$.

Example 2:

An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time $t \geq 0$ with

$\frac{dx}{dt} = 2 \sin(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(3, 5)$.

a) Find the x -coordinate of the position of the object at time $t = 4$.

b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -6 . Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

c) Find the magnitude of the velocity vector at time $t = 2$.

d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t - 1$. Find the acceleration vector of the object at time $t = 4$.