

§10.2—Partial Fraction Decomposition

In section [7.5](#) (BC), we learned how to integrate rational functions by using partial fraction decomposition using the Heaviside “Cover Up” method. This works great for denominators that factor into non-repeating, linear factors (as long as the degree of the numerator is less than the degree of the denominator, in which case one must long divide first).

In this section, we will learn the proper way to decompose a rational function into its partial fractions when the factors include either repeated factors or are non-linear, or both!

Example 1:

Under the ever-watchful eye of Oliver Heaviside, review/learn the Heaviside method for integrating the following integral:

$$\int \frac{7x-13}{x^2-2x-3} dx$$



There is a more general method which we will now explore:

Method 1: Equating Coefficients**Example 2:**

Evaluate $\int \frac{1-3x}{2x^2-3x+1} dx$

Method 2: Plug-n-Chug**Example 3:**

Evaluate $\int \frac{4x^2 + 13x - 12}{x^3 + x^2 - 6x} dx$

Any of the above integrals could appear on a BC Calculus exam, but there are other types out there that are worth exploring. Let's look at some.

Type I: Repeating Linear Factor**Example 4:**

Evaluate $\int \frac{3x^2 + 4x + 5}{(x-1)(x^2 + x - 2)} dx$

Example 5:

Evaluate $\int \frac{3x^2 + 16x + 14}{(x+3)^3} dx$

Type II: Irreducible Quadratic Factor

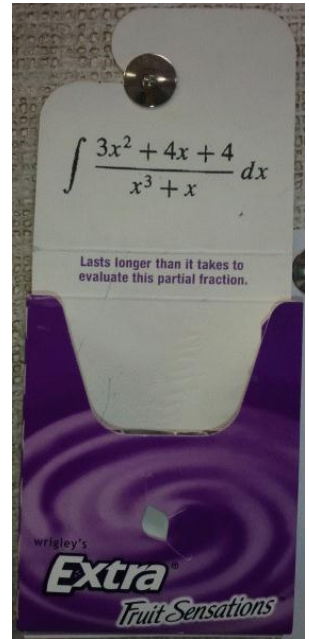
If the denominator has a repeating quadratic factor, then the partial fraction decomposition will have a term of the form $\frac{Ax + B}{ax^2 + bx + c}$

Example 6:

On a pack of Extra Fruit Sensations gum package was written the following integral:

$$\int \frac{3x^2 + 4x + 4}{x^3 + x} dx$$

Beneath this, the gum maker claims that the gum's flavor "[l]asts longer than it takes to evaluate this partial fraction." Obviously this gum was not designed for an AP Calculus student, as a student of this caliber requires his gum to hold its flavor for much, much, much longer. To prove my point, we will evaluate this integral using partial fraction decomposition. Let's get chewing!!



Note 1: If the denominator has a repeated irreducible factor, similar to the repeated linear case, each power of the factor will get its own term in the decomposition.

Note 2: For other types of irreducible factor, the numerator will be of a form that is one degree smaller than the numerator.

Note 3: Of course, if the degree of the numerator is greater than or equal to the degree of the denominator, one must use long division first, then decompose the resulting rational remainder.

Bonus Example 7:

Set up, but do not decompose, a decomposition for the following rational expression.

$$\frac{x^4 + 3x - 2}{(x^2 + 1)^3 (x^2 - 8x + 16)}$$

Bonus Example 8:

Evaluate $\int \frac{x^5 - 2x^4 + x^3 + x + 5}{x^3 - 2x^2 + x - 2} dx$

Bonus Example 9:

Evaluate $\int \frac{2x^2 + x + 11}{(x+3)(x^2 + 2x + 10)} dx$