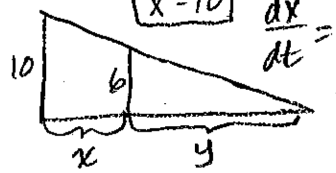
(1) $C = 2\pi r$ $A = \pi r^2$

$\frac{dC}{dr} = 2\pi \frac{dr}{dt}$ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

want $\frac{dA}{dt} = 2 \cdot \frac{dC}{dt}$ for some r

$\cancel{2\pi} r \cancel{\frac{dr}{dt}} = 2\left(\cancel{2\pi} \cancel{\frac{dr}{dt}}\right)$

$\boxed{r = 2}$ $\boxed{B}$

(2) 

$\boxed{x = 10}$ $\frac{dx}{dt} = +4$

$\frac{dy}{dt}$ = how fast shadow length is changing

$\frac{10}{6} = \frac{x+y}{y}$

$5y = 3x + 3y$

$2y = 3x$

$y = \frac{3}{2}x$

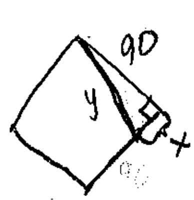
$\frac{d}{dt}$: $\frac{dy}{dt} = \frac{3}{2}\frac{dx}{dt}$

$\frac{d(x+y)}{dt} = \frac{dx}{dt} + \frac{dy}{dt}$ = how fast tip of shadow is moving

at $x = 10$ (or always in this case)

$\frac{dy}{dx} = \frac{3}{2}(4) = 6$ ft/sec

so $\frac{d(x+y)}{dt} = 4 + 6 = \boxed{10 \text{ ft/sec}}$ $\boxed{B}$

(3) 

$\boxed{x = 30 \text{ ft}}$

$\frac{dx}{dt} = -25$ ft/sec

$\frac{dy}{dt}$ = rate of change of dist. to 2nd base = ?

$x^2 + 90^2 = y^2$

$\frac{d}{dt}$: $2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$x\frac{dx}{dt} = y\frac{dy}{dt}$

when $x = 30$,

$y = \sqrt{90^2 + 30^2}$

$y = \sqrt{9000}$ ft

$y = 30\sqrt{10}$ ft

when $x = 30$:

$\cancel{30}(-25) = \cancel{30}\sqrt{10} \cdot \frac{dy}{dt}$

$\frac{dy}{dt} = -\frac{25}{\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right) = \frac{-25\sqrt{10}}{10} = -\frac{5}{2}\sqrt{10}$

*so when $x = 30$ ft, the runner's distance to 2nd base is decreasing by $\boxed{\frac{5}{2}\sqrt{10} \text{ ft/sec}}$ $\boxed{A}$

(4) $5x^3 + 6y^3 = xy$, $\boxed{\left(\frac{1}{11}, \frac{1}{11}\right)}$ $\frac{dy}{dt} = +5$, $\frac{dx}{dt} = ?$

$\frac{d}{dt}$: $15x^2\frac{dx}{dt} + 18y^2\frac{dy}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$

at $\left(\frac{1}{11}, \frac{1}{11}\right)$: $\frac{15}{121}\frac{dx}{dt} + \frac{18}{121}(5) = \frac{1}{11}\left(\frac{dx}{dt}\right) + \left(\frac{1}{11}\right)(5)$

$\left(\frac{15}{121} - \frac{1}{11}\right)\frac{dx}{dt} = \frac{5}{11} - \frac{90}{121}$

$\frac{4}{121}\frac{dx}{dt} = \frac{-35}{121}$

$\frac{dx}{dt} = -\frac{35}{4}$ units/sec $\boxed{F}$

so x is decreasing by $\frac{35}{4}$ units/sec

(5) 

$\frac{d\theta}{dt} = 2$, $\frac{dx}{dt} = ?$, $\boxed{x = 4}$

$\sin\theta = \frac{x}{5}$

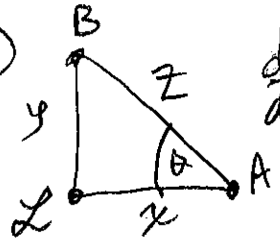
when $x = 4$, $y = 3$ and $\cos\theta = \frac{3}{5}$

$x = 5\sin\theta$

$\frac{d}{dt}$: $\frac{dx}{dt} = 5\cos\theta \cdot \frac{d\theta}{dt}$

when $x = 4$: $\frac{dx}{dt} = 5\left(\frac{3}{5}\right)(2) = \boxed{6 \text{ ft/hr}}$ $\boxed{D}$

(6)



$\frac{dx}{dt} = -15, \ \frac{dy}{dt} = 10$

(a) $z^2 = x^2 + y^2$

$\boxed{x = 4, y = 3}$

$\boxed{z = 5 \text{ km}}$

(b) $\frac{dz}{dt} = ?$

$\frac{d}{dt}: \cancel{2}z\frac{dz}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$

when $x=4$: $5\left(\frac{dz}{dt}\right) = 4(-15) + (3)(10)$

$\frac{dz}{dt} = \frac{-60+30}{5} = \boxed{-6 \text{ km/hr}}$

(c) $\frac{d\theta}{dt} = ?$

$\tan\theta = \frac{y}{x}$

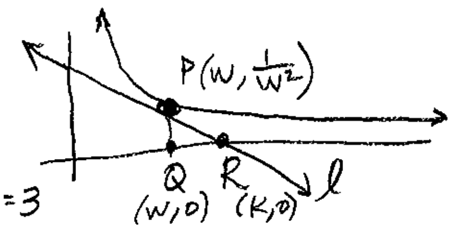
$\frac{d}{dt}: \sec^2\theta \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$

when $x=4$: $\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{4(10) - 3(-15)}{4^2}$

$\frac{d\theta}{dt} = \frac{\cancel{16}}{25}\left(\frac{40+45}{\cancel{16}}\right) = \frac{85}{25}$

$\boxed{\frac{d\theta}{dt} = \frac{17}{5} \text{ rad/hr}}$

(7) $y = \frac{1}{x^2}$



(a) $k = ?$ when $w = 3$

k is the x-int of the tangent line

$y(3) = \frac{1}{9}, \ P\left(3, \frac{1}{9}\right)$

$y'(x) = \frac{-2}{x^3}, \ y'(3) = \frac{-2}{27}$

eq of l: $l(x) = \frac{1}{9} - \frac{2}{27}(x-3)$

x-int (let $l(x)=0$): $0 = \frac{1}{9} - \frac{2}{27}(x-3)$

$\frac{2}{27}(x-3) = \frac{1}{9}$

$x - 3 = \frac{3}{2}$

$x = 3 + \frac{3}{2} = \boxed{\frac{9}{2} = k}$

(b) In general at any pt P

$P\left(w, \frac{1}{w^2}\right), \ y'(w) = \frac{-2}{w^3}$

so $l(x) = \frac{1}{w^2} - \frac{2}{w^3}(x - w)$

at $(k, 0)$: $0 = \frac{1}{w^2} - \frac{2}{w^3}(k - w)$

$\frac{2}{w^3}(k-w) = \frac{1}{w^2}$

$k - w = \frac{w}{2}$

$k = w + \frac{w}{2}$

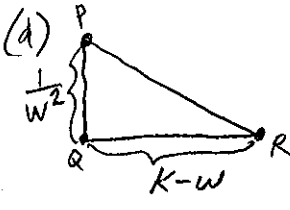
$\boxed{k = \frac{3w}{2}}$

(c) $\frac{dw}{dt} = 7, \ \boxed{w = 5}, \ \frac{dk}{dt} = ?$

$\frac{d}{dt}: \frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt}$

when $w=5$:
(or for all w in this case) $\frac{dk}{dt} = \frac{3}{2}(7) = \boxed{\frac{21}{2}}$

(d)



Area $= A = \frac{1}{2}bh$

$A = \frac{1}{2}(k-w)\left(\frac{1}{w^2}\right)$

$A = \frac{1}{2}\left(\frac{3}{2}w - w\right)\left(\frac{1}{w^2}\right)$

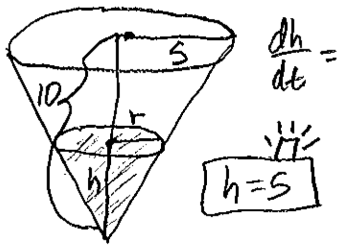
$A = \frac{1}{4}w\left(\frac{1}{w^2}\right) = \frac{1}{4w}$

$\frac{d}{dt}: \frac{dA}{dt} = -\frac{1}{4w^2} \cdot \frac{dw}{dt}$

when $w=5$: $\frac{dA}{dt} = -\frac{1}{100}(7) = \frac{-7}{100} < 0$

so Area is decreasing at this moment.

(8)



$\frac{dh}{dt} = \frac{-3}{10}$, by similar triangles

$\frac{10}{5} = \frac{h}{r}$

$\boxed{h = 2r \text{ and } r = \frac{1}{2}h}$

$\boxed{h=5}$

(a) $V = \frac{\pi}{3}r^2h$ so $V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$ or $V = \frac{\pi}{3}r^2(2r)$

$V = \frac{\pi}{12}h^3$ or $V = \frac{2\pi}{3}r^3$

so $V(h) = \frac{\pi}{12}h^3$, $V(5) = \frac{\pi}{12}(5^3) = \boxed{\frac{125\pi}{12} \text{ cm}^3}$

(b) $\frac{dV}{dt} = ?$ when $\boxed{h=5}$

$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\left.\frac{dV}{dt}\right|_{h=5} = \frac{\pi}{4}(5^2)\left(-\frac{3}{10}\right) = \frac{-75\pi}{40} = \boxed{\frac{-15\pi}{8} \text{ cm}^3/\text{hr}}$

(c) Show $\frac{dV}{dt} = K \cdot \overbrace{\pi r^2}^{\text{area of circle}}$ for some K

$V(r) = \frac{2\pi}{3}r^3$ since $r = \frac{1}{2}h$

$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$ $\boxed{\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}}$

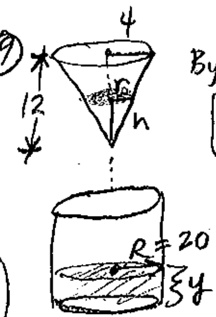
so $\frac{dV}{dt} = \cancel{2}\pi r^2\left(\frac{1}{\cancel{2}}\frac{dh}{dt}\right)$

$\frac{dV}{dt} = \frac{dh}{dt} \cdot \pi r^2$

so $\boxed{K = \frac{dh}{dt} = -\frac{3}{10}}$

*K = constant of proportionality

(9)



Area of base of cylinder

$400\pi = \pi R^2$

$R^2 = 400$

$\boxed{R = 20 \text{ ft}}$

(a constant as the cylider fills!)

By similar $\triangle$s

$\boxed{\frac{12}{4} = \frac{h}{r}}$

$\frac{dh}{dt} = h - 12$

(a) $V = \frac{\pi}{3}r^2h$, $r = \frac{1}{3}h$

so $V(h) = \frac{\pi}{3}\left(\frac{1}{3}h\right)^2 h$

$\boxed{V(h) = \frac{\pi}{27}h^3}$

(b) $\boxed{h=3}$ $\frac{dV}{dt} = ?$

$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{9}h^2(h-12)$

$\left.\frac{dV}{dt}\right|_{h=3} = \frac{\pi}{\cancel{9}}(\cancel{9})(-9)$

$\boxed{\frac{dV}{dt} = -9\pi \text{ ft}^3/\text{min}}$

*when h = 3, the volume of water in the conical tank is decreasing by 9π ft^3 per min.

(c) Let $\mathcal{V}$ be the volume of cyclindrical tank

$\mathcal{V} = \pi R^2 y$, so $\mathcal{V} = 400\pi y$

$\boxed{\frac{d\mathcal{V}}{dt} = 400\pi\frac{dy}{dt}}$

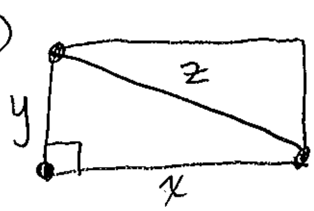
*the rate at which the cylindrical tank fills is the same as the rate at which the conical tank empties.

So $\frac{d\mathcal{V}}{dt} = -\frac{dV}{dt} = 9\pi \text{ ft}^3/\text{min}$

So at h = 3: $9\pi = 400\pi\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9\cancel{\pi}}{400\cancel{\pi}} = \boxed{\frac{9}{400} \text{ ft/min}}$

(10)



$\frac{dz}{dt} = 1$, $\frac{dx}{dt} = 3\frac{dy}{dt}$ so $\frac{dy}{dt} = \frac{1}{3}\frac{dx}{dt}$ $\boxed{\begin{matrix} x=4 \\ y=3 \end{matrix}}$ $\boxed{\frac{dx}{dt} = ?}$

primary eq

$x^2 + y^2 = z^2$ so $z = \sqrt{16+9} \Rightarrow \boxed{z=5}$

$\frac{d}{dt}: \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

$\boxed{x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}}$

when $x=4$: $4\left(\frac{dx}{dt}\right) + 3\left(\frac{1}{3}\frac{dx}{dt}\right) = 5(1)$

$5\frac{dx}{dt} = 5$

$\boxed{\frac{dx}{dt} = 1}$

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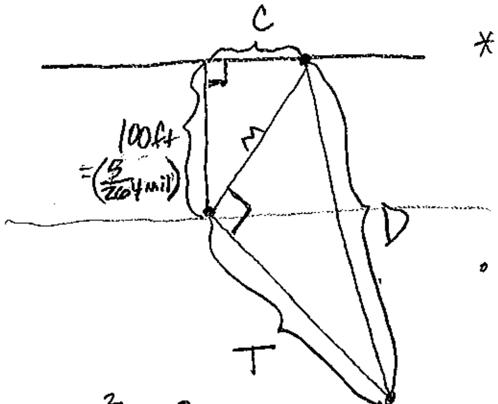
(11)



*we will put all units in miles and hours

- $100ft\left(\frac{1 mile}{5280 ft}\right) = \frac{5}{264}$ mile = Bridge height
- $10sec\left(\frac{1 min}{60 sec}\right)\left(\frac{1 hr}{60 min}\right) = \frac{1}{360}$ hr
- Let C = distance calculus car has travelled in 10 sec

  $\boxed{\frac{dC}{dt} = +45}$ is car's velocity
- Let T = distance train has travelled in 10 sec

  $\boxed{\frac{dT}{dt} = 60}$ is train's velocity
- Let D = distance betwixt Calculus Car and train

  $\boxed{\frac{dD}{dt} = ?}$

- $C^2 + \left(\frac{5}{264}\right)^2 = M^2$
- $M^2 + T^2 = D^2$

so $\boxed{C^2 + \left(\frac{5}{264}\right)^2 + T^2 = D^2}$

After 10 sec $= \frac{1}{360}$ hr: $C = (45)\left(\frac{1}{360}\right) = \frac{1}{8}$

Dist = (Rate)(Time) $T = (60)\left(\frac{1}{360}\right) = \frac{1}{6}$

$D = \sqrt{\left(\frac{1}{8}\right)^2 + \left(\frac{5}{264}\right)^2 + \left(\frac{1}{6}\right)^2} = 0.209...$ store as "D"

$\frac{d}{dt}: \cancel{2}C\frac{dC}{dt} + \cancel{2}T\frac{dT}{dt} = \cancel{2}D\frac{dD}{dt}$

$C\frac{dC}{dt} + T\frac{dT}{dt} = D\frac{dD}{dt}$

$$\boxed{\frac{dD}{dt} = \frac{C\frac{dC}{dt} + T\frac{dT}{dt}}{D}}$$

$$\left.\frac{dD}{dt}\right|_{\text{after 10 seconds}} = \frac{\left(\frac{1}{8}\right)(45) + \left(\frac{1}{6}\right)(60)}{0.209...}$$

$$= \boxed{74.69199049 \text{ mph}}$$

* So after 10 seconds, the distance between Calculus Car and the train is increasing by 74.691 miles per hour