① $f'(x) = 12x^2 - 6x + 1,\ f(1) = 5,\ f(0) =$

$f(x) = 4x^3 - 3x^2 + x + C$

at $(1,5)$: $5 = 4 - 3 + 1 + C$

$\boxed{C = 3}$

so $f(x) = 4x^3 - 3x^2 + x + 3$

so $\boxed{f(0) = 3}$ $\boxed{B}$

② $g'(x) = \dfrac{5x^2 + 4x + 5}{\sqrt{x}}$

$g'(x) = \dfrac{5x^2}{x^{1/2}} + \dfrac{4x}{x^{1/2}} + \dfrac{5}{x^{1/2}}$

$g'(x) = 5x^{3/2} + 4x^{1/2} + 5x^{-1/2}$

$g(x) = 5\left(\tfrac{2}{5}\right)x^{5/2} + 4\left(\tfrac{2}{3}\right)x^{3/2} + 5(2)x^{1/2} + C$

$g(x) = 2x^{5/2} + \tfrac{8}{3}x^{3/2} + 10x^{1/2} + C$

factor out least powers

$\boxed{g(x) = 2x^{1/2}\left(x^2 + \tfrac{4}{3}x + 5\right) + C}$ $\boxed{B}$

③ $f''(t) = 2(3t+1),\ f'(1) = 3,\ f(1) = 5$

$f''(t) = 6t + 2$

$f'(t) = 3t^2 + 2t + C$

for $f'(1) = 3$: $3 = 3 + 2 + C$, $\boxed{C = -2}$

so $f'(t) = 3t^2 + 2t - 2$

$f(t) = t^3 + t^2 - 2t + C$

for $f(1) = 5$: $5 = 1 + 1 - 2 + C$, $\boxed{C = 5}$

so $\boxed{f(t) = t^3 + t^2 - 2t + 5}$ $\boxed{E}$

④ Antiderivatives of $f(x) = \sin x \cos x$.

Plan: take derivative of each I, II, and III

I. $F_1(x) = \frac{1}{2}(\sin x)^2,\ F_1'(x) = (\sin x)' \cdot \cos x$ ✓

II. $F_2(x) = -\frac{1}{4}\cos 2x = -\frac{1}{4}(\cos^2 x - \sin^2 x)$ (Double-Angle)

$F_2'(x) = -\frac{1}{4}(2\cos x(-\sin x) - 2\sin x \cos x)$

$= -\frac{1}{4}(-4\sin x \cos x)$

$= \sin x \cos x$ ✓

III. $F_3(x) = -\frac{1}{2}(\cos x)^2,\ F_3'(x) = -(\cos x)'(-\sin x)$

$= \sin x \cos x$ ✓

So $\boxed{D}$ $\boxed{\text{I, II, \& III}}$

⑤ $a(t) = 8 - 8t,\ v(0) = 12$

$v(t) = 8t - 4t^2 + C$

for $v(0) = 12$: $12 = C$

so $v(t) = 8t - 4t^2 + 12$

$x(t) = \text{position} = -\frac{4}{3}t^3 + 4t^2 + 12t + C$

Maximize $x(t)$

so $x'(t) = v(t) = -4t^2 + 8t + 12 = 0$

$-4(t^2 - 2t - 3) = 0$

$-4(t-3)(t+1) = 0$

So $t = 3$ or $t \neq -1$

since $x''(3) = a(3) = 8 - 8(3) < 0$,

$t = 3$ maximizes $x(t)$

so $\boxed{\text{3 seconds}}$ $\boxed{C}$

⑥ (a) $\int (\sqrt{x^3} + 2x + 1)\,dx$

$= \int (x^{3/2} + 2x + 1)\,dx$

$= \boxed{\tfrac{2}{5}x^{5/2} + x^2 + x + C}$

(b) $\int \left(\dfrac{x^3 + 2x - 3}{x^4}\right)dx$

$= \int \left(\dfrac{x^3}{x^4} + \dfrac{2x}{x^4} - \dfrac{3}{x^4}\right)dx$

$= \int \left(\dfrac{1}{x} + 2x^{-3} - 3x^{-4}\right)dx$

$= \boxed{\ln|x| - x^{-2} + x^{-3} + C}$

6 (c) $\int (2t^2-1)^2\,dt$

$= \int (4t^4-4t^2+1)\,dt$

$= \boxed{\frac{4}{5}t^5-\frac{4}{3}t^3+t+C}$

(d) $\int (\theta^2+\sec^2\theta-\csc\theta\cot\theta)\,d\theta$

$= \boxed{\frac{1}{3}\theta^3+\tan\theta+\csc\theta+C}$

(e) $\int \left(\frac{\cos x}{1-\cos^2 x}\right)dx$

$= \int \left(\frac{\cos x}{\sin^2 x}\right)dx$

$= \int \left(\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}\right)dx$

$= \int (\csc x\cot x)\,dx$

$= \boxed{-\csc x+C}$

(f) $\int (\cos x+3^x)\,dx$

$= \boxed{\sin x+\frac{1}{\ln 3}\cdot 3^x+C}$

7 (a) $f'(x)=4x,\ f(0)=6$

$f(x)=2x^2+C$

for $f(0)=6$: $6=2(0^2)+C$

$C=6$

so $\boxed{f(x)=2x^2+6}$

(b) $h'(t)=8t^3+5,\ h(1)=-4$

$h(t)=2t^4+5t+C$

for $h(1)=-4$: $-4=2+5+C$

$C=-11$

so $\boxed{h(t)=2t^4+5t-11}$

(c) $f''(x)=2,\ f'(2)=5,\ f(2)=10$

$f'(x)=2x+C,\quad 5=2(2)+C$

$C=1$

so $f'(x)=2x+1$

$f(x)=x^2+x+C,\quad 10=2^2+2+C$

$C=4$

so $\boxed{f(x)=x^2+x+4}$

(d) $f''(x)=x^{-3/2},\ f'(4)=2,\ f(0)=0$

$f'(x)=-2x^{-1/2}+C;\ 2=\frac{-2}{\sqrt{4}}+C,\ \boxed{C=3}$

so $f'(x)=-2x^{-1/2}+3$

$f(x)=-4x^{1/2}+3x+C;\ 0=0+0+C,\ \boxed{C=0}$

so $\boxed{f(x)=-4\sqrt{x}+3x}$

(e) $f''(x)=\sin x,\ f'(0)=1,\ f(0)=6$

$f'(x)=-\cos x+C,\quad 1=-\cos(0)+C$

$C=2$

so $f'(x)=-\cos x+2$

$f(x)=-\sin x+2x+C,\quad 6=-\sin(0)+2(0)+C$

$C=6$

so $\boxed{f(x)=-\sin x+2x+6}$