Worksheet 5.2—Slope Fields
Show all work when applicable.

Short Answer and Free Response:

Draw a slope field for each of the following differential equations.

1. \( \frac{dy}{dx} = x + 1 \)

2. \( \frac{dy}{dx} = 2y \)

3. \( \frac{dy}{dx} = x + y \)

4. \( \frac{dy}{dx} = 2x \)

5. \( \frac{dy}{dx} = y - 1 \)

6. \( \frac{dy}{dx} = -\frac{y}{x} \)
For 7 – 12, match each slope field with the equation that the slope field could represent.

7. (E)  
8. (G)  

9. (C)  
10. (A)  

11. (H)  
12. (B)  

(A) \( y = 1 \)

(B) \( y = x \)

(C) \( y = x^2 \)

(D) \( y = \frac{1}{6}x^3 \)

(E) \( y = \frac{1}{x^2} \)

(F) \( y = \sin x \)

(G) \( y = \cos x \)

(H) \( y = \ln|x| \)
For 13 – 16, match the slope fields with their differential equations.

13. \( \mathbf{A} \)

14. \( \mathbf{C} \)

15. \( \mathbf{B} \)

16. \( \mathbf{C} \)

(A) \( \frac{dy}{dx} = \frac{1}{2}x + 1 \)

(B) \( \frac{dy}{dx} = x - y \)

(C) \( \frac{dy}{dx} = y \)

(D) \( \frac{dy}{dx} = -\frac{x}{y} \)

17. The calculator-drawn slope field for the differential equation \( \frac{dy}{dx} = x + y \) is shown in the figure below.

(a) Sketch the solution curve through the point \( (0,1) \).

(b) Sketch the solution curve through the point \( (-3,0) \).

(c) Approximate \( y(-3.1) \) using the equation of the tangent line to \( y = f(x) \) at the point \( (-3,0) \).

\[
\left. \frac{dy}{dx} \right|_{(-3,0)} = -3 + 0 = -3
\]

\[
\mathcal{L}(x) = 0 - 3(x + 3)
\]
18. Consider the differential equation \( \frac{dy}{dx} = 2y - 4x \).

(a) The slope field for the differential equation is shown below. Sketch the solution curve that passes through the point \((0,1)\) and sketch the solution curve that goes through the point \((0,-1)\).

(b) There is a value of \(b\) for which \( y = 2x + b \) is a solution to the differential equation. Find this value of \(b\). Justify your answer.

(c) Let \(g\) be the function that satisfies the given differential equation with the initial condition \( g(0) = 0 \).

It appears from the slope field that \(g\) has a local maximum at the point \((0,0)\). Using the differential equation, prove analytically that this is so.
Multiple Choice:

19. Given the following slope field (with **equilibrium solutions**, meaning the slopes are zero and a the existence of a horizontal asymptote on the solution graph, at \( y = 0 \) and \( y = 1 \)), find the matching differential equation.

\[
\begin{align*}
(A) \quad & \frac{dy}{dx} = y(y-1) \\
(B) \quad & \frac{dy}{dx} = y(1-y) \\
(C) \quad & \frac{dy}{dx} = \frac{1}{y(1-y)} \\
(D) \quad & \frac{dy}{dx} = 1 - e^{y(1-y)} \\
(E) \quad & \frac{dy}{dx} = \frac{y}{y-1}
\end{align*}
\]

20. A slope field for the differential equation \( \frac{dy}{dx} = 42 - y \) will show

(A) a line with slope \(-1\) and \(y\)-intercept of 42 
(B) a vertical asymptote at \( x = 42 \) 
(C) a horizontal asymptote at \( y = 42 \) 
(D) a family of parabolas opening downward 
(E) a family of parabolas opening to the left

21. For which of the following differential equations will a slope field show nothing but negative slopes in the fourth quadrant?

\[
\begin{align*}
(A) \quad & \frac{dy}{dx} = -\frac{x}{y} \\
(B) \quad & \frac{dy}{dx} = xy + 5 \\
(C) \quad & \frac{dy}{dx} = xy^2 - 2 \\
(D) \quad & \frac{dy}{dx} = \frac{x^3}{y^2} \\
(E) \quad & \frac{dy}{dx} = \frac{y}{x^2} - 3
\end{align*}
\]

22. Which of the following differential equations would produce the slope field shown below?

\[
\begin{align*}
(A) \quad & \frac{dy}{dx} = y - |x| \\
(B) \quad & \frac{dy}{dx} = |y| - x \\
(C) \quad & \frac{dy}{dx} = |y - x| \\
(D) \quad & \frac{dy}{dx} = |y + x| \\
(E) \quad & \frac{dy}{dx} = |y| - |x|
\end{align*}
\]
23. Which of the following differential equations would produce the slope field shown below?

\[ (A) \frac{dy}{dx} = y - 3x \quad (B) \frac{dy}{dx} = y - \frac{x}{3} \quad (C) \frac{dy}{dx} = y + \frac{x}{3} \quad (D) \frac{dy}{dx} = x + \frac{y}{3} \quad (E) \frac{dy}{dx} = x - \frac{y}{3} \]

24. AP 2010B-5 (No Calculator)

Consider the differential equation \( \frac{dy}{dx} = \frac{x + 1}{y} \).

(a) On the axes provided at right, sketch a slope field for the given differential equation at the twelve points indicated, and for \(-1 < x < 1\), sketch the solution curve that passes through the point \((0, -1)\).

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the \(xy\)-plane for which \(y \neq 0\). Describe all points in the \(xy\)-plane, \(y \neq 0\), for which \(\frac{dy}{dx} = -1\).

\[ \frac{x + 1}{y} = -1 \quad \frac{x + 1}{-y} = -1 \quad y = -x - 1 \]

So, \(\frac{dy}{dx} = -1\) for all points along the line \(y = -x - 1\) for \(y \neq 0\).

(c) Find the particular solution \(y = f(x)\) to the given differential equation with the initial condition \(f(0) = -2\).

\[ \frac{dy}{dx} = \frac{x + 1}{y} \]

\[ y \ dy = (x + 1) \ dx \]

\[ \int y \ dy = \int (x + 1) \ dx \]

\[ \frac{1}{2} y^2 = \frac{1}{2} x^2 + x + C \]

\[ y^2 = x^2 + 2x + C \]

\[ y = \pm \sqrt{x^2 + 2x + C} \]

At \((0, -2)\):

\[ -2 = -\sqrt{0 + 0 + C} \]

\[ y = C \]

So, \(y = -\sqrt{x^2 + 2x + C}\)
25. AP 2006-5 (No Calculator)

Consider the differential equation \( \frac{dy}{dx} = \frac{1+y}{x} \), where \( x \neq 0 \).

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(b) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(-1) = 1 \) and state its domain.

\[
\begin{align*}
\frac{dy}{dx} &= \frac{1+y}{x} \\
\int \frac{1+y}{dx} &= \int \frac{1}{x} dx \\
\ln|1+y| &= \ln|x| + c \\
|1+y| &= e^{\ln|x|+c} \\
1+y &= C \cdot e^{\ln|x|} \\
1+y &= C |x| \\
y &= C |x| - 1
\end{align*}
\]
26. AP 2005-6 (No Calculator)

Consider the differential equation \( \frac{dy}{dx} = \frac{-2x}{y} \).

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) Let \( y = f(x) \) be the particular solution to the differential equation with the initial condition \( f(1) = -1 \). Write an equation for the line tangent to the graph of \( f \) at \((1, -1)\) and use it to approximate \( f(1.1) \).

\[
\begin{align*}
\frac{dy}{dx} &= \frac{-2x}{y} \\
\frac{dy}{dx} \bigg|_{(1,-1)} &= \frac{-2}{-1} = 2
\end{align*}
\]

\( \mathcal{L}(x) = -1 + 2(x - 1) \)

\[
\begin{align*}
\mathcal{L}(1.1) &\approx f(1.1) = -1 + 2(0.1) \\
&= -1 + 0.2 \\
&= -0.8
\end{align*}
\]

(c) Find the particular solution \( y = f(x) \) to the given differential equation with the initial condition \( f(1) = -1 \).

\[
\begin{align*}
\frac{dy}{dx} &= \frac{-2x}{y} \\
y \ dy &= -2x \ dx \\
\int y \ dy &= \int -2x \ dx \\
\frac{1}{2} y^2 &= -x^2 + C \\
y^2 &= -2x^2 + C \\
y &= \pm \sqrt{-2x^2 + C}
\end{align*}
\]

@ \((1, -1)\):

\[
\begin{align*}
y &= \pm \sqrt{-2 \cdot 1 + C} \\
1 &= -2 + C \\
C &= 3
\end{align*}
\]

\( y = -\sqrt{2x^2 + 3} \)

*For what it's worth:

\[
\begin{align*}
f(1.1) &= -\sqrt{-2.2 + 3} \\
&= -\sqrt{0.8} \\
&= -0.894
\end{align*}
\]

& since \( \mathcal{L}(1.1) > f(1.1) \),

\( f(x) \) is concave up

for \( x \in (1, 1.1) \)