① $x = e^{2t}$, $y = \sin(3t)$

$\frac{dx}{dt} = 2e^{2t}$, $\frac{dy}{dt} = 3\cos 3t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \boxed{\frac{3\cos(3t)}{2e^{2t}}}$

② $x = \cos^3 t$, $y = \sin^2 t$, $0 \le t \le \frac{\pi}{2}$

$\text{Length} = \int_0^{\pi/2} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$

$x'(t) = 3\cos^2 t(-\sin t) = -3\sin t\cos^2 t$, $y'(t) = 2\sin t\cos t \overset{\text{or}}{=} 2\sin 2t$ (Double angle !!)

so $\boxed{L = \int_0^{\pi/2} \sqrt{9\sin^2 t\cos^4 t + 4\sin^2 t\cos^2 t}\, dt}$

③ $x = t^3 - t^2 - 1$, $y = t^4 + 2t^2 - 8t$

$\frac{dx}{dt} = 3t^2 - 2t$, $\frac{dy}{dt} = 4t^3 + 4t - 8$

vertical tangent when $\frac{dx}{dt} = 0$, $\frac{dy}{dt} \neq 0$

$\frac{dy}{dx} = \frac{\neq 0}{0}$

so $3t^2 - 2t = 0$

$t(3t-2) = 0$

$t = 0$, $t = \frac{2}{3}$

$\left.\frac{dy}{dt}\right|_{t=0} = 8 \neq 0$

$\left.\frac{dy}{dt}\right|_{t=\frac{2}{3}} = \frac{32}{27} + \frac{8}{3} - 8 \neq 0$

So the curve has vertical tangents for $t = 0$ and $t = \frac{2}{3}$

④ $x(t) = 3t^2 - 4t + 2$, $y(t) = t^3 - 4t$

$x'(t) = 6t - 4$, $y'(t) = 3t^2 - 4$

$\left.\frac{dy}{dx}\right|_{t=1} = \frac{3(1^2) - 4}{6(1) - 4} = \frac{-1}{2} = m$

$(x(1), y(1)) = (3 - 4 + 2, 1 - 4) = (1, -3)$

so tangent line eq. is

$\boxed{y = -3 - \frac{1}{2}(x-1)}$

⑤ $x(t) = e^t + 1$, $y = 2e^{2t}$ *eliminate the parameter

$e^t = x - 1$

$t = \ln(x-1)$ plug in

$y = 2e^{2\ln(x-1)}$

$y = 2e^{\ln(x-1)^2}$

$\boxed{y = 2(x-1)^2}$

⑥ $x = \cos(5t)$, $y = t^3$

$x'(t) = -5\sin 5t$, $y'(t) = 3t^2$

$\vec{v}(t) = \langle -5\sin 5t, 3t^2 \rangle$

$\vec{v}(2) = \langle -5\sin 10, 12 \rangle$

Speed @ $t = 2$ is $\|\vec{v}(2)\| = \sqrt{25\sin^2 10 + 144}$

$\boxed{\approx 12.304}$

⑦ $x(t) = \frac{1}{3}(t-2)^3 + 4$, $y(t) = t^2 - 4t + 4$

$\vec{v}(t) = \langle (t-2)^2, 2t - 4 \rangle$

(a) $\vec{v}(1) = \langle 1, -2 \rangle$

$\|\vec{v}(1)\| = \sqrt{1 + 4} = \boxed{\sqrt{5}} \approx 2.236$

(b) $\text{Dist} = \int_0^1 \sqrt{(t-2)^4 + (2t-4)^2}\, dt$

$\approx \boxed{3.815 \text{ or } 3.816}$

(c) Particle at rest when $\vec{v}(t) = 0$, or when

$x'(t) = (t-2)^2 = 0$ & $y'(t) = 2t - 4 = 0$

$\underline{t = 2}$

$2t = 4$

$\underline{t = 2}$

So particle is at rest when $\boxed{t = 2}$.

(8) $t \geq 0$, $\frac{dx}{dt} = x'(t) = 1 + \tan(t^2)$, $\frac{dy}{dt} = y'(t) = 3e^{\sqrt{t}}$

$\vec{v}(t) = \langle 1 + \tan(t^2), 3e^{\sqrt{t}} \rangle$

$\boxed{\vec{a}(t) = \vec{v}'(t) = \left\langle 2t\sec^2(t^2), \frac{3}{2\sqrt{t}} e^{\sqrt{t}} \right\rangle}$

$\vec{v}(5) = \langle 1 + \tan(25), 3e^{\sqrt{5}} \rangle$

speed at $t = 5$ is $\|\vec{v}(5)\| = \sqrt{(1+\tan 25)^2 + (3e^{\sqrt{5}})^2}$

$= \boxed{28.082 \text{ or } 28.083}$

(9) $x(t) = t + \cos t$, $y(t) = 3t + 2\sin t$, $0 \leq t \leq \pi$

$y = 3t + 2\sin t = 5$

when $t = 1.079\ldots = A$ ← store as A

From calculator

$\vec{v}(A) = \langle x'(A), y'(A) \rangle$

$= \boxed{\langle 0.1185, 3.944 \rangle}$

(10) $\frac{dx}{dt} = x'(t) = 2\sin(t^3)$, $\frac{dy}{dt} = y'(t) = \cos(t^2)$, $0 \leq t \leq 4$

at $t = 1$, $(x(1), y(1)) = (3, 4)$

(a) tangent line at (3,4).

$\left.\frac{dy}{dx}\right|_{(3,4)} = \frac{y'(1)}{x'(1)} = \frac{\cos 1}{2\sin 1} = 0.321 = A$

eq: $\boxed{y = 4 + 0.321(x-3)}$

(b) speed at $t = 2$ is $\|\vec{v}(2)\|$

$= \sqrt{(2\sin 8)^2 + (\cos 4)^2} = \boxed{2.083} = B$

(c) Dist $= \int_0^1 \sqrt{(2\sin(t^3))^2 + (\cos(t^2))^2}\, dt$

$= \boxed{1.126}$

(d) Position at $t = 2$

$(x(2), y(2)) = \left(3 + \int_1^2 x'(t)\,dt,\ 4 + \int_1^2 y'(t)\,dt\right)$

$= \boxed{(3.436, 3.556)}$

(11) $\frac{dx}{dt} = \cos(t^2)$, $\frac{dy}{dt} = \sin(t^3)$, at $t = 0$, $(x(0), y(0)) = (4, 7)$

$(x(2), y(2)) = \left(4 + \int_0^2 \cos(t^2)\,dt,\ 7 + \int_0^2 \sin(t^3)\,dt\right) = (4.461, 7.451)$

or $\boxed{\langle 4.461, 7.452 \rangle}$ $\boxed{D}$

(12) $x = \sin(2t),\ y = \cos(5t)$

speed at $t=2$ is $\sqrt{(2\cos(2\cdot 2))^2 + (-5\sin(5\cdot 2))^2} \approx \boxed{3.0179..}$ $\boxed{B}$

(13) $x(t) = t^3 - t^2 - 1,\ y(t) = t^4 + 2t^2 - 8t$

$x'(t) = 3t^2 - 2t\ ,\ y'(t) = 4t^3 + 4t - 8$

vert tangent when $\frac{dy}{dx} = \frac{\neq 0}{0}$, or when $x'(t) = 0,\ y'(t) \neq 0$

$3t^2 - 2t = 0$

$t(3t-2) = 0$

when $t=0,\ t=\frac{2}{3}$

$y'(0) = -8 \neq 0$

$y'\left(\frac{2}{3}\right) = 4\left(\frac{8}{27}\right) + 4\left(\frac{2}{3}\right) - 8 \neq 0$

So the curve has vertical tangents at $\boxed{t=0 \text{ and } t=\frac{2}{3}}$ $\boxed{C}$

(14) $\vec{s}(t) = \langle t^2, t\rangle,\ 0 \le t \le 4\ ,\ \vec{s}'(t) = \vec{v}(t) = \langle 2t, 1\rangle$

$\boxed{\text{Dist} = \int_0^4 \sqrt{4t^2+1}\, dt}$ $\boxed{D}$