Worksheet P.5—Domain, Range, and Symmetry

Show all work. No Calculator

1. Find the x- and y-intercepts and domain, then sketch the graph and find the range.
   
   (a) \( f(x) = \sqrt{2-x} \)
   
   (b) \( g(x) = \sqrt{4-x^2} \)

   (c) \( f(t) = \frac{|t-3|}{t-3} \)

   (d) \( h(m) = \begin{cases} (m-1)^2, & m \geq 1 \\ 3m-3, & m < 1 \end{cases} \)

2. Sketch the following piecewise functions, then find the domain and range of each.

   (a) \( f(x) = \begin{cases} 3-x, & x \leq 1 \\ 2x, & 1 < x \end{cases} \)

   (b) \( g(x) = \begin{cases} 2, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases} \)

   (c) \( h(t) = \begin{cases} t^2, & t < 0 \\ t^3, & 0 \leq t \leq 1 \\ 2t-1, & t > 1 \end{cases} \)
3. Write a piecewise function for the given graphs
(a) ____________________________ (b) ____________________________

Multiple Choice

_____ 4. Which of the following defines a function \( f \) for which \( f(-x) = -f(x) \)?

(A) \( f(x) = x^2 \)  (B) \( f(x) = \sin x \)  (C) \( f(x) = \cos x \)  (D) \( f(x) = \log x \)  (E) \( f(x) = e^x \)

_____ 5. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) \( y = \frac{x+1}{x} \)  (B) \( y = -x^5 + 3x \)  (C) \( y = x^4 - 2x^2 + 6 \)  (D) \( y = (x-1)^3 + 1 \)  (E) \( y = (x^2 + 1)^2 - 1 \)
4. If \( F(x) = x^{-2/3} (x-2)^{2/3} + x^{1/3} (x-2)^{-1/3} \), find the domain of \( F \).

(A) \( D_F : \{x | x \neq 0\} \)  (B) \( D_F : \{x | x > 0\} \)  (C) \( D_F : \{x | 0 \leq x \leq 2\} \)

(D) \( D_F : \{x | x \neq 0 \text{ and } x \neq 2\} \)  (E) \( D_F : \{x | x \text{ is a real number}\} \)

5. The domain of the function defined by \( g(x) = \ln(x^2 - 4) \) is the set of all real numbers \( x \) such that

(A) \( |x| < 2 \)  (B) \( |x| \leq 2 \)  (C) \( |x| > 2 \)  (D) \( |x| \geq 2 \)  (E) \( x \) is a real number

6. The graph of \( y^2 = x^2 + 9 \) is symmetric to which of the following?

I. The \( x \)-axis
II. The \( y \)-axis
III. The origin

(A) I only  (B) II only  (C) III only  (D) I and II only  (E) I, II, and III

7. What is the domain of the function \( f \) given by \( f(x) = \frac{\sqrt{x^2 - 4}}{x - 3} \) ?

(A) \( \{x : x \neq 3\} \)  (B) \( \{x : |x| \leq 2\} \)  (C) \( \{x : |x| \geq 2\} \)

(D) \( \{x : |x| \geq 2 \text{ and } x \neq 3\} \)  (E) \( \{x : x \geq 2 \text{ and } x \neq 3\} \)
8. Let \( f \) and \( g \) be odd functions. If \( p, r, \) and \( s \) are nonzero functions defined as follows, which must be odd?

I. \( p(x) = f(g(x)) \)

II. \( r(x) = f(x) + g(x) \)

III. \( s(x) = f(x)g(x) \)

(A) I only       (B) II only       (C) I and II only       (D) II and III only       (E) I, II, and III

9. If the function \( f \) is continuous for all real numbers and if \( f(x) = \frac{x^2 - 4}{x + 2} \) when \( x \neq -2 \), then \( f(-2) = \)

(A) \(-4\)       (B) \(-2\)       (C) \(-1\)       (D) \(0\)       (E) \(2\)

10. Let \( f \) be the function defined by \( f(x) = \begin{cases} x^3 & \text{for } x \leq 0, \\ x & \text{for } x > 0. \end{cases} \) Which of the following statements about \( f \) is true?

(A) \( f \) is an odd function       (B) There is a break in the graph of \( f \) at \( x = 0 \)       (C) \( f \) has no \( x \)-intercepts

(D) \( f \) is monotonic increasing       (E) \( f(|x|) = |x| \)
11. Find the domain of the function \( f(x) = \frac{\sqrt{x+1}}{x-5} \).

(A) \( D_f : \{ x \mid x \geq 1 \} \)  
(B) \( D_f : \{ x \mid x < 1, x \neq -5 \} \)  
(C) \( D_f : \{ x \mid x \leq -1, x \neq -5 \} \)  
(D) \( D_f : \{ x \mid x > -1, x \neq -5 \} \)  
(E) \( D_f : \{ x \mid x \leq 1 \} \)  
(F) \( D_f : \{ x \mid x \geq -1, x \neq -5 \} \)

12. Which of the following functions has the following graph of \( x \in [-6,6], x \neq 1 \)

(A) \( f(x) = - \frac{x^2 - 1}{|x+1|} \)  
(B) \( f(x) = \frac{|x^2 - 1|}{x-1} \)  
(C) \( f(x) = - \frac{|x^2 - 1|}{x-1} \)  
(D) \( f(x) = \frac{x^2 - 1}{|x+1|} \)  
(E) \( f(x) = \frac{x^2 - 1}{|x-1|} \)  
(F) \( f(x) = - \frac{x^2 - 1}{|x-1|} \)

13. Which of the following gives the domain of \( f(x) = \frac{x}{\sqrt{9-x^2}} \)?

(A) \( x \neq \pm 3 \)  
(B) \( (-3,3) \)  
(C) \( [-3,3] \)  
(D) \( (-\infty,-3) \cup (3,\infty) \)  
(E) \( (3,\infty) \)
14. Which of the following gives the range of \( f(x) = 1 + \frac{1}{x-1} \)?

(A) \((\infty, 1) \cup (1, \infty)\)  
(B) \(y \neq 1\)  
(C) all real numbers  
(D) \((\infty, 0) \cup (0, \infty)\)  
(E) \(y \neq 0\)

15. Which of the following gives the range of \( y = 4 - 2^{-x} \)?

(A) \((-\infty, 2)\)  
(B) \((-\infty, 4)\)  
(C) \([-4, \infty)\)  
(D) \((-\infty, 4)\)  
(E) all reals

16. Which of the following gives the domain of \( f(x) = 3 - \ln(x+2) \)?

(A) \(x \neq 2\)  
(B) \((\infty, \infty)\)  
(C) \((-\infty, \infty)\)  
(D) \([-1.9, \infty)\)  
(E) \((0, \infty)\)

17. The domain of the function \( f(x) = \ln(x^2 - x - 6) \) is the set of all real numbers \( x \) such that

(A) \(x > 0\)  
(B) \(-2 \leq x \leq 3\)  
(C) \(x \geq -2\) or \(x \geq 3\)  
(D) \(x < -2\) or \(x > 3\)  
(E) \(-2 < x < 3\)

18. The domain of \( y = \sqrt{(x-1)(x-2)} \) is

(A) \(|x| < 2\)  
(B) \([1, 2)\)  
(C) \(|x| > 2\)  
(D) \((\infty, 1] \cup [2, \infty)\)  
(E) \([1, 2]\)