Worksheet 1.1—Limits & Continuity

**Short Answer:** Show all work. Unless stated otherwise, no calculator permitted.

1. Explain in your own words what is meant by the equation \( \lim_{x \to 2} f(x) = 4 \). Is it possible for this statement to be true and yet \( f(2) = 5 \)? Explain. What graphical manifestation would \( f(x) \) have at \( x = 2 \)? Sketch a possible graph of \( f(x) \).

2. Explain what it means to say that \( \lim_{x \to 1^-} f(x) = 3 \) and \( \lim_{x \to 1^+} f(x) = 6 \). What graphical manifestation would \( f(x) \) have at \( x = 1 \)? Sketch a possible graph of \( f(x) \).

3. Explain the meaning of each of the following, then sketch a possible graph of a function exhibiting the indicated behavior.
   - (a) \( \lim_{x \to -2^-} f(x) = \infty \)
   - (b) \( \lim_{x \to 3^+} g(x) = -\infty \).

4. For \( f(x) = \frac{x^2 + x - 20}{x^2 - 16} \), algebraically determine the following:
   - (a) \( f(4) \)
   - (b) \( \lim_{x \to 4^-} f(x) \)
   - (c) \( \lim_{x \to 4^+} f(x) \)
   - (d) \( \lim_{x \to 4} f(x) \)
   - (e) \( \lim_{x \to 4} f(x) \)
   - (f) \( \lim_{x \to 0^-} f(x) \)
   - (g) \( \lim_{x \to 1} f(x) \)
   - (h) \( \lim_{x \to -1} f(x) \)
5. Using the definition of continuity, determine whether the graph of \( f(x) = \frac{x^2 + x}{x^3 + 2x^2 - 3x} \) is continuous at the following. Justify.

(a) \( x = 0 \)  
(b) \( x = 1 \)  
(c) \( x = 2 \)

6. For \( f(x) = \begin{cases} -x^2, & x < 0 \\ \sqrt{x}, & x > 0 \end{cases} \), algebraically determine the following:

(a) \( f(0) \)  
(b) \( \lim_{x \to 0^-} f(x) \)  
(c) \( \lim_{x \to 0^+} f(x) \)  
(d) \( \lim_{x \to 0} f(x) \)  
(e) continuity of \( f \) at \( x = 0 \). Justify.

7. Evaluate each of the following continuous functions at the indicated \( x \)-value:

(a) \( \lim_{\theta \to \frac{11\pi}{6}} \sin \theta = \)  
(b) \( \lim_{x \to 6} 2^x = \)  
(c) \( \lim_{x \to 0} \left( 57x^{85} - 2x^{45} + 100x^{11} - 99999x + 5 \right) = \)

8. Evaluate each of the following:

(a) \( \lim_{x \to -\frac{\pi}{2}} \tan x = \)  
(b) \( \lim_{x \to \frac{\pi}{2}^-} \tan x = \)  
(c) \( \lim_{x \to \frac{\pi}{2}^+} \tan x = \)

(d) \( \lim_{x \to -5^-} \frac{-2}{x+5} = \)  
(e) \( \lim_{x \to -5^+} \frac{-2}{x+5} = \)  
(f) \( \lim_{x \to -5} \frac{-2}{x+5} = \)
9. For the function \( f \) whose graph is given at below, evaluate the following, if it exists. If it does not exist, explain why.

(a) \( \lim_{x \to 3} f(x) = \)

(b) \( \lim_{x \to 7} f(x) = \)

(c) \( \lim_{x \to 4} f(x) = \)

(d) \( \lim_{x \to 9^-} f(x) = \)

(e) \( \lim_{x \to 9^+} f(x) = \)

(f) \( \lim_{x \to 9} f(x) = \)

(g) What are the equations of the vertical asymptotes?

10. A patient receives a 150-mg injection of a drug every four hours. The graph at right shows the amount \( C(t) \) of the drug in the bloodstream after \( t \) hours.

Approximate \( \lim_{t \to 12^-} C(t) \) and \( \lim_{t \to 12^+} C(t) \), then explain in a complete sentence the significance/meaning of these one-sided limits in terms of the injections at \( t = 12 \) hours.
11. **(Calculator Permitted)** Sketch the graph of the function \( f(x) = \frac{1}{1 + 2^{1/x}} \) in the space below, then evaluate each, if it exists. If it does not exist, explain why. Name the type and location of any discontinuity.

\[
\begin{align*}
(a) \lim_{x \to 0^-} f(x) = \\
(b) \lim_{x \to 0^+} f(x) = \\
(c) \lim_{x \to 0} f(x) = \\
(d) f(0) =
\end{align*}
\]

12. Using the definition of continuity at a point, discuss the continuity of the following function. Justify.

\[
\begin{align*}
f(x) = \begin{cases} 
2 - x, & x < -1 \\
x, & -1 \leq x < 1 \\
(x - 1)^2, & x \geq 1
\end{cases}
\end{align*}
\]

13. For \( f(x) = \begin{cases} 
3ax - b, & x < 1 \\
5, & x = 1 \\
2a\sqrt{x} + b, & x > 1
\end{cases} \), find the values of \(a\) and \(b\) such that \( f(x) \) is continuous at \( x = 1 \). Show the work that leads to your answer.
14. **(Calculator permitted)** Fill in the table for the following function, then use the numerical evidence (to 3 decimal places) to evaluate the indicated limit. (Be sure you’re in radian mode)

\[ f(x) = \frac{\sin(3x)}{x} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-0.1)</th>
<th>(-0.01)</th>
<th>(-0.001)</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
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</tbody>
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Based on the numeric evidence above, \( \lim_{x \to 0} f(x) = \)

**Multiple Choice:** Put the Capital Letter of the correct answer in the blank to the left of the number. Be sure to show any work/analysis.

_____ 15. \( \lim_{x \to 0} \left( 1 - \frac{1}{x} \right) = \)

(A) 1 \hspace{1cm} (B) 2 \hspace{1cm} (C) \(-\infty\) \hspace{1cm} (D) 0 \hspace{1cm} (E) \(\infty\)

_____ 16. Find \( \lim_{x \to 1} f(x) \) if \( f(x) = \begin{cases} 
3 - x, & x \neq 1 \\
1, & x = 1
\end{cases} \)

(A) 2 \hspace{1cm} (B) 1 \hspace{1cm} (C) \(\frac{3}{2}\) \hspace{1cm} (D) 0 \hspace{1cm} (E) DNE

_____ 17. For which of the following does \( \lim_{x \to 4} f(x) \) exist?

I. \hspace{4cm} II. \hspace{4cm} III.

|\( y \)| |\( x \)| |\( y \)| |
|---|---|---|---|
|graph of \( f(x) \)| |graph of \( f(x) \)| |graph of \( f(x) \)|

(A) I only \hspace{1cm} (B) II only \hspace{1cm} (C) III only \hspace{1cm} (D) I and II only \hspace{1cm} (E) I and III only

_____ 18. If \( f(x) = \begin{cases} 
\ln x, & 0 < x \leq 2 \\
x^2 \ln 2, & 2 < x \leq 4
\end{cases} \), then \( \lim_{x \to 2} f(x) \) is

(A) \(\ln 2\) \hspace{1cm} (B) \(\ln 8\) \hspace{1cm} (C) \(\ln 16\) \hspace{1cm} (D) 4 \hspace{1cm} (E) nonexistent
Use the graph of \( f(x) \) above to answer questions 19 - 22.

_____ 19. \( \lim_{{x \to 7}} f(x) = \)

(A) 1 \hspace{1cm} (B) 2 \hspace{1cm} (C) -1 \hspace{1cm} (D) 4 \hspace{1cm} (E) DNE

_____ 20. \( \lim_{{x \to 0^-}} f(x) = \)

(A) 1 \hspace{1cm} (B) 2 \hspace{1cm} (C) -1 \hspace{1cm} (D) 4 \hspace{1cm} (E) DNE

_____ 21. \( \lim_{{x \to 2}} f(x) = \)

(A) 2 \hspace{1cm} (B) 3 \hspace{1cm} (C) -1 \hspace{1cm} (D) 4 \hspace{1cm} (E) DNE

_____ 22. Which of the following regarding \( f(x) \) at \( x = 5 \) true?

I. \( \lim_{{x \to 5^-}} f(x) = 3 \)

II. \( \lim_{{x \to 5^+}} f(x) = f(5) \)

III. \( f(x) \) is continuous at \( x = 5 \)

(A) I only \hspace{1cm} (B) II only \hspace{1cm} (C) I and II only \hspace{1cm} (D) II and III only \hspace{1cm} (E) I, II, and III
23. If \( f(x) = \begin{cases} ae^x + b, & x < 0 \\ 4, & x = 0 \\ bx - 2a, & x > 0 \end{cases} \), then the value of \( b \) that makes \( f(x) \) continuous at \( x = 0 \) is

(A) 2  (B) -2  (C) 4  (D) 6  (E) no such value exists

24. If \( f(x) = \frac{1}{x-2} \) and \( \lim_{x \to (-k+1)} f(x) \) does not exist, then \( k = \)

(A) 2  (B) 3  (C) 1  (D) -2  (E) -1

25. The function \( f(x) = \begin{cases} \frac{x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \)

(A) is continuous for all \( x \)
(B) has a removable point discontinuity at \( x = 0 \)
(C) has a non-removable oscillation discontinuity at \( x = 0 \)
(D) has an non-removable infinite discontinuity at \( x = 0 \)
(E) has a non-removable jump discontinuity at \( x = 0 \)

26. If \( f(x) = \begin{cases} \frac{x^2 - x}{2x}, & x \neq 0 \\ k, & x = 0 \end{cases} \) is continuous at \( x = 0 \), then \( k = \)

(A) -1  (B) -\frac{1}{2}  (C) 0  (D) \frac{1}{2}  (E) 1