Worksheet 1.2—Properties of Limits

Show all work. Unless stated otherwise, no calculator permitted.

Short Answer

1. Given that \( \lim_{x \to a} f(x) = -3 \), \( \lim_{x \to a} g(x) = 0 \), \( \lim_{x \to a} h(x) = 8 \), for some constant \( a \), find the limits that exist. If the limit does not exist, explain why.

(a) \( \lim_{x \to a} [f(x) + h(x)] = \)  
(b) \( \lim_{x \to a} [f(x)]^2 = \)  
(c) \( \lim_{x \to a} \sqrt{h(x)} = \)  
(d) \( \lim_{x \to a} \frac{1}{f(x)} = \)  

(e) \( \lim_{x \to a} \frac{f(x)}{h(x)} = \)  
(f) \( \lim_{x \to a} \frac{g(x)}{f(x)} = \)  
(g) \( \lim_{x \to a} \frac{f(x)}{g(x)} = \)  
(h) \( \lim_{x \to a} \frac{2f(x)}{h(x) - f(x)} = \)
2. The graphs of \( f \) and \( g \) are given below. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.

(a) \( \lim_{x \to 2} [f(x) + g(x)] = \)

(b) \( \lim_{x \to 1} [2f(x) - 3g(x)] = \)

(c) \( \lim_{x \to 0} f(x)g(x) = \)

(d) \( \lim_{x \to 1} \frac{f(x)}{g(x)} = \)

(e) \( \lim_{x \to 2} x^3f(x) = \)

(f) \( \lim_{x \to 1^-} f(g(x)) = \)
3. The graphs of the functions \( f(x) = x \), \( g(x) = -x \), and 
\[ h(x) = x \cos\left(\frac{50\pi}{x}\right) \]
on the interval \(-1 \leq x \leq 1\) are given at right.

Use the Squeeze Theorem to find 
\[ \lim_{x \to 0} x \cos\left(\frac{50\pi}{x}\right) \]. Justify.

4. If \( 1 \leq f(x) \leq x^2 + 2x + 2 \) for all \( x \), find 
\[ \lim_{x \to 1} f(x) \]. Justify.

5. If \( -3\cos(\pi x) \leq f(x) \leq x^3 + 2 \), evaluate 
\[ \lim_{x \to 1} f(x) \]. Justify
Multiple Choice

6. Suppose \( 2 \leq f(x) \leq (1-x)^2 + 2 \) for all \( x \neq 1 \) and that \( f(1) \) is undefined. What is \( \lim_{x \to 1} f(x) \)?

(A) 3  
(B) 2  
(C) 4  
(D) \( \frac{5}{2} \)  
(E) 1

Use the graphs of the function \( f(x) \) and \( g(x) \) shown above to answer questions 7 – 9.

7. \( \lim_{x \to 2^-} \left( \frac{f(x)}{g(x)} \right) = \)

(A) 1  
(B) -1  
(C) 2  
(D) -2  
(E) DNE

8. \( \lim_{x \to 3^-} f(g(x)) = \)

(A) 0  
(B) -1  
(C) 2  
(D) 1  
(E) DNE

9. \( g(1) + \lim_{x \to 1^+} x \cdot f(x) = \)

(A) 0  
(B) -1  
(C) 2  
(D) 1  
(E) DNE