Name____ Date_______________________ Period______

Worksheet 1.5—Continuity on Intervals & IVT

Show all work. No Calculator (unless stated otherwise)

Short Answer

1. Let $f(x)=\begin{cases} x^2, & x\le 1 \\ x^2-2x-1, & 1<x<3 \\ 4, & x\ge 3 \end{cases}$.

(a) Sketch a graph of $f(x)$.

(b) Based on the function above, list the largest intervals on $x\in(-\infty,\infty)$ for which $f(x)$ is continuous.

(c) Find a number b such that $f(x)$ is continuous in $(-\infty,b]$ but not in $(-\infty,b+1)$.

(d) Find all numbers a and b such that $f(x)$ is continuous in (a,b) but not in $(a,b]$.

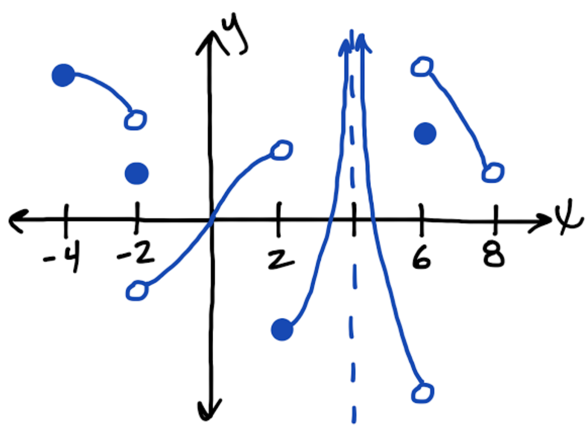
(e) Find the least number a such that $f(x)$ is continuous in $[a,\infty)$.

2. A toy car travels on a straight path. During the time interval $0 \le t \le 60$ seconds, the toy car's velocity *v*, measured in feet per second, is a continuous function. Selected values are given below.

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-10	-15	-10	-7	-5	0	13

For $0 < t < 60$, must there be a time *t* when $v(t) = -2$? Justify.

3. The graph of *f* is given below, and has the property of $\lim_{x \to 4^-} f(x) = \infty$



(a) Can the IVT be used to prove that $f(x) = 31415926$ somewhere on the interval $x \in [2, 4]$? Why or why not? Will, in fact, $f(x) = 31415926$ on this interval?

(b) State the largest intervals for which the given graph of f is continuous.

4. For the function $f(x)=\begin{cases}(x-2)^2, & x=4\\ 5, & 4<x\le 10\end{cases}$. Find $f(4)$ and $f(10)$. Does the IVT guarantee a *y*-value *u* on $4\le x\le 10$ such that $f(4)<u<f(10)$? Why or why not. Sketch the graph of $f(x)$ for added visual proof.

5. If f and g are continuous functions with $f(3)=5$ and $\lim_{x\to 3}\left[2f(x)-g(x)\right]=4$, find $g(3)$.

6. Determine the values of *x* for which the function $f(x)=\begin{cases}\frac{1}{x}, & x<1\\ x^2, & 1\le x<2\\ \sqrt{8x}, & 2<x\le 8\\ 8.0001, & x>8\end{cases}$ is continuous.

7. Use the IVT to show that there is a solution to the given functions on the given intervals. Be sure to test your hypothesis, show numeric evidence, and write a concluding statement. Use your calculator to find the actual solution value correct to three decimal places.

(a) $\cos x = x,\ (0,1)$

(b) $\ln x = e^{-x},\ (1,2)$

8. Mr. Wenzel is mountain climbing with Mr. Korpi. They leave the base of Mount BBB at 7:00 A.M. and take a single trail to the top of the mountain, arriving at the summit at 7:00 P.M. where they spend a sleepless night dodging bears and lightning bolts in their heads. The next morning, they wearily leave the summit at 7:00 A.M. and travel down the same path they came up the day before, arriving at the base of the mountain at 7:00 P.M. Will there be a point along the trail where Mr. Wenzel and Mr. Korpi will be standing at exactly the same time of day on consecutive days? Why or why not?

9. The functions *f* and *g* are continuous for all real numbers. The table below gives values of the functions at selected values of *x*. The function *h* is given by $h(x) = g(f(x)) + 2$.

x	$f(x)$	$g(x)$
1	3	4
3	9	–10
5	7	5
7	11	25

Explain why there must be a value w for $1 < w < 5$ such that $h(w) = 0$

10. The functions *f* and *g* are continuous for all real numbers. The function *h* is given by $h(x) = f(g(x)) - x$. The table below gives values of the functions at selected values of *x*. Explain why there must be a value of u for $1 < u < 4$ such that $h(u) = -1$.

x	1	2	3	4
$f(x)$	0	8	–3	6
$g(x)$	3	4	1	2

Multiple Choice

_____ 11. Let $g(x)$ be a continuous function. Selected values of g are given in the table below.

x	3	5	6	9	10
$g(x)$	2	5	-1	4	0

What is the fewest number of times the graph of $g(x)$ will intersect $y=1$ on the closed interval $[3,10]$?

(A) None (B) One (C) Two (D) Three (E) Four

______ 12. Let $h(x)$ be a continuous function. Selected values of h are given in the table below.

x	2	3	4	5	7
$h(x)$	2	5	k	4	3

For which value of k will the equation $h(x)=\frac{2}{3}$ have **at least two solutions** on the closed interval $[2,7]$?

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{7}{9}$ (D) $\frac{2}{3}$ (E) $\frac{11}{18}$

_____ 13. If $f(x)=\begin{cases} x+1, & x\le 1 \\ 3+ax^2, & x>1 \end{cases}$, then $f(x)$ is continuous for all x if $a=$

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 0 (E) -2

_____ 14. If $f(x)=\begin{cases} \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\ne 2 \\ k, & x=2 \end{cases}$, and if f is continuous at $x=2$, then $k=$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

_____ 15. Let f be the function defined by the following.

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

_____ 16. Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3)=-1$ and $f(6)=3$, then the Intermediate Value Theorem guarantees that

(A) $f(0)=0$

(B) The slope of the graph of f is $\frac{4}{9}$ somewhere between -3 and 6

(C) $-1\le f(x)\le 3$ for all x between -3 and 6

(D) $f(c)=1$ for at least one c between -3 and 6

(E) $f(c)=0$ for at least one c between -1 and 3

_____ 17. Let f be the function given by $f(x)=\dfrac{(x-1)(x^2-4)}{x^2-a}$. For what **positive** values of a is f continuous for all real numbers x?

(A) None (B) 1 only (C) 2 only (D) 4 only (E) 1 and 4 only

_____ 18. If f is continuous on $[-4,4]$ such that $f(-4)=11$ and $f(4)=-11$, then which must be true?

(A) $f(0)=0$ (B) $\lim_{x\to 2} f(x)=8$ (C) There is at least one $c\in[-4,4]$ such that $f(c)=8$

(D) $\lim_{x\to 3} f(x)=\lim_{x\to -3} f(x)$ (E) It is possible that f is not defined at $x=0$