Name_____ Date_____ Period_____

Worksheet 2.1—Tangent Line Problem

Show all work. No calculator permitted, except when stated.

Short Answer

1. Find the derivative function, $f'(x)$, for each of the following using the limit definition.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

(a) $f(x)=2x^2+3x-4$ (b) $f(x)=\frac{3}{x-1}$ (c) $f(x)=\sqrt{x-2}$

2. Find the slope of the tangent lines to the graphs of the following functions at the indicated points. Use the ***alternate form.***

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

(a) $f(x)=3-2x$ at $(-1,5)$ (b) $g(x)=5-x^2$ at $x=2$

3. Find the equation of the tangent line, in Taylor Form: $y = y_1 + m(x - x_1)$, for $g(x) = x^2 + 1$ at $(2,5)$. Use the ***modified form*** to find $g'(2)$.

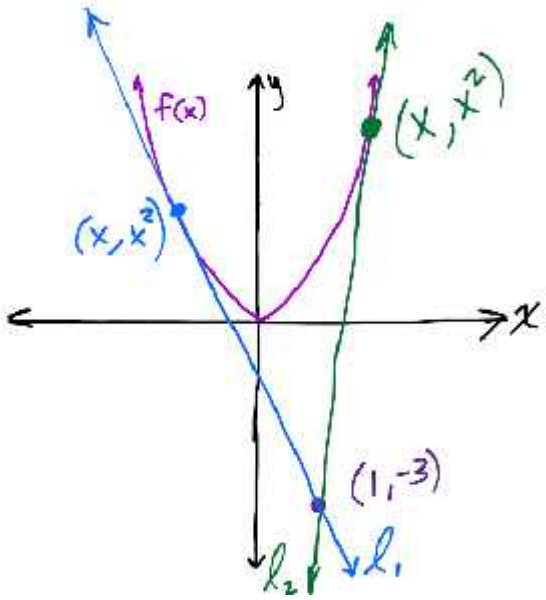
$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

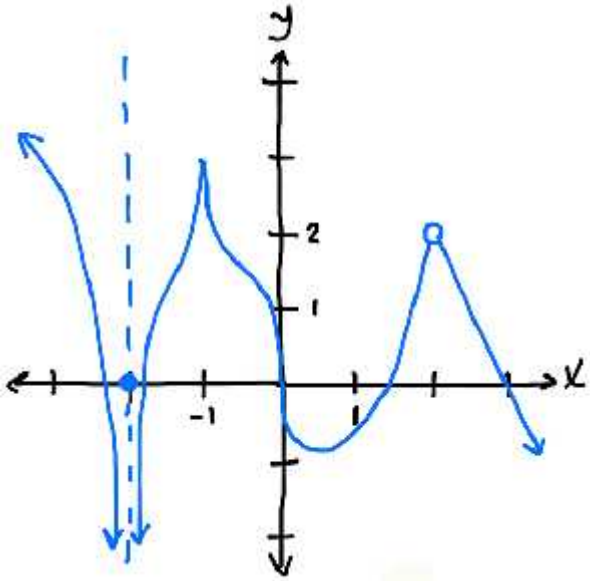
4. Find the equation of the tangent line, in Taylor Form: $y = y_1 + m(x - x_1)$, for $y = \sqrt{x} - 1$ at $c = 9$. Use the ***alternate form*** to find $y'(9)$.

$$y'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

5. Find an equation of the line that is tangent to $f(x) = x^3$ and parallel to the line $3x - y + 1 = 0$. Remember, parallel lines have the same slope, but different base camps.

6. Find the equations of the two lines, ℓ_1 and ℓ_2, that are tangent to the graph of $f(x) = x^2$ if each pass through the point $(1, -3)$, as shown at right. Hint: equate two different expressions for finding the slope of a line. Solve the resulting equation.





7. The graph of a function $f(x)$ is show above. For which value(s) of x is the graph of $f(x)$ not differentiable. In each case, explain why not.

8. For each of the following, the limit represents $f'(c)$ for a function $f(x)$ and a number $x=c$. Find both f and c.

(a) $\lim_{h\to 0}\dfrac{\left[5-3(1+h)\right]-2}{h}$

(b) $\lim_{h\to 0}\dfrac{(-2+h)^3+8}{h}$

(c) $\lim_{x\to 6}\dfrac{-x^2+36}{x-6}$

(d) $\lim_{x\to 9}\dfrac{2\sqrt{x}-6}{x-9}$

9. Using the alternate form, determine whether each of the following function is differentiable at the indicated point. Show the work that leads to your answer.

(a) $f(x)=\begin{cases}5-4x, & x\le 0\\ -2x^2, & x>0\end{cases}$ at $x=0$

(b) $f(x)=\begin{cases}(x-1)^3, & x\le 1\\ (x-1)^2, & x>1\end{cases}$ at $x=1$

10. **True or False. If false, explain why or give a counterexample.**

(a) The slope of the tangent line to the differentiable function f at the point $(2, f(2))$ is $\dfrac{f(2+h)-f(2)}{h}$.

(b) If a function is continuous at a point, then that function is differentiable at that point.

(c) If a function's slopes from both the right and the left at a point are the same, then that function is differentiable at that point.

(d) If a function is differentiable at a point, then that function is continuous at that point.

11. Using your **calculator** to zooooooom in, determine if $h(x)=\sqrt{x^2+0.0001}+0.99$ is locally linear at $x=0$. Give a reason for your answer.

Multiple Choice

_____ 12. A function will fail to be differentiable at all of the following except

(A) A vertical asymptote (B) A removable discontinuity (C) A cusp
(D) A vertical tangent line (E) A horizontal tangent line

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & x\neq 2\\ 1, & x=2\end{cases}$$

_____ 13. Let f be the function defined above. Which of the following statements about f are true?

I. $\lim\limits_{x\to 2} f(x)$ exists
II. f is continuous at $x=2$
III. f is differentiable at $x=2$

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

_____ 14. Let f be a differentiable function such that $f(2)=1$ and $f'(2)=4$. Let $T(x)$ be the equation of the tangent line to $f(x)$ at $x=2$. What is the value of $T(1.9)$?

(A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

_____ 15. Let f be a function such that $\lim_{h\to 0}\frac{f(7+h)-f(7)}{h}=5$. Which of the followign must be true?

I. f is continuous at $x=7$

II. f is differentiable at $x=7$

III. The derivative of f is differentiable at $x=7$

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

_____ 16. At $x=4$, the function given by $h(x)=\begin{cases} x^2, & x\le 4 \\ 4x, & x>4 \end{cases}$ is

(A) Undefined

(B) Continuous but not differentiable

(C) Differentiable but not continuous

(D) Neither continuous nor differentiable

(E) Both continuous and differentiable