

Name _____ Date _____ Period _____

Worksheet 2.1—Tangent Line Problem

Show all work. No calculator permitted, except when stated.

Short Answer

1. Find the derivative function, $f'(x)$, for each of the following using the limit definition.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

(a) $f(x) = 2x^2 + 3x - 4$

(b) $f(x) = \frac{3}{x-1}$

(c) $f(x) = \sqrt{x-2}$

2. Find the slope of the tangent lines to the graphs of the following functions at the indicated points. Use the *alternate form*.

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

(a) $f(x) = 3 - 2x$ at $(-1, 5)$

(b) $g(x) = 5 - x^2$ at $x = 2$

3. Find the equation of the tangent line, in Taylor Form: $y = y_1 + m(x - x_1)$, for $g(x) = x^2 + 1$ at $(2, 5)$. Use the *modified form* to find $g'(2)$.

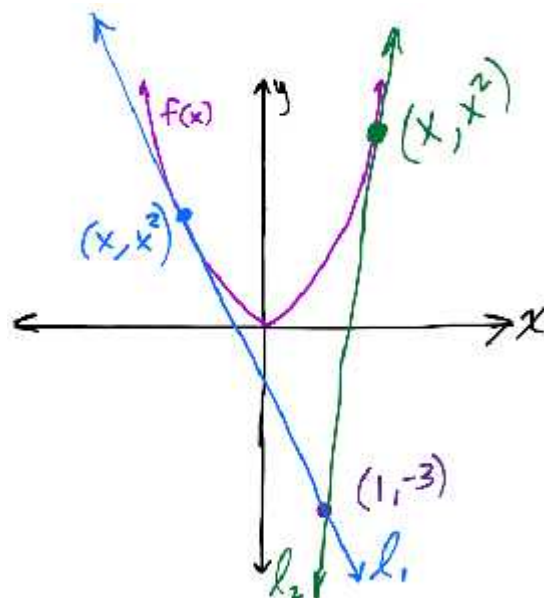
$$f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

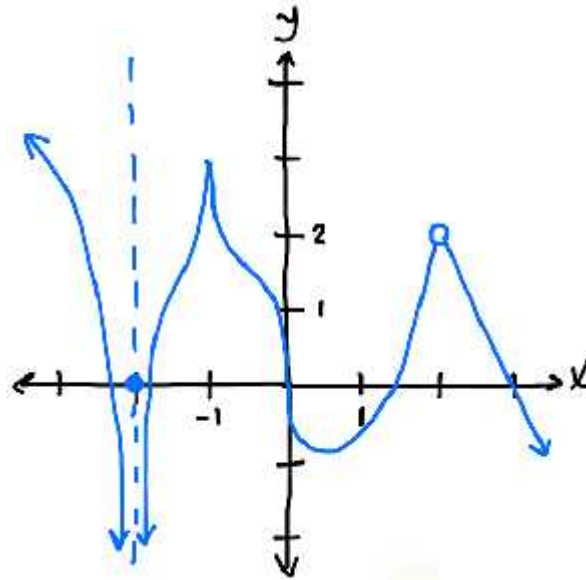
4. Find the equation of the tangent line, in Taylor Form: $y = y_1 + m(x - x_1)$, for $y = \sqrt{x} - 1$ at $c = 9$. Use the *alternate form* to find $y'(9)$.

$$y'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

5. Find an equation of the line that is tangent to $f(x) = x^3$ and parallel to the line $3x - y + 1 = 0$. Remember, parallel lines have the same slope, but different base camps.

6. Find the equations of the two lines, l_1 and l_2 , that are tangent to the graph of $f(x) = x^2$ if each pass through the point $(1, -3)$, as shown at right. Hint: equate two different expressions for finding the slope of a line. Solve the resulting equation.





7. The graph of a function $f(x)$ is show above. For which value(s) of x is the graph of $f(x)$ not differentiable. In each case, explain why not.

8. For each of the following, the limit represents $f'(c)$ for a function $f(x)$ and a number $x = c$. Find both f and c .

$$(a) \lim_{h \rightarrow 0} \frac{[5 - 3(1+h)] - 2}{h}$$

$$(b) \lim_{h \rightarrow 0} \frac{(-2+h)^3 + 8}{h}$$

$$(c) \lim_{x \rightarrow 6} \frac{-x^2 + 36}{x - 6}$$

$$(d) \lim_{x \rightarrow 9} \frac{2\sqrt{x} - 6}{x - 9}$$

9. Using the alternate form, determine whether each of the following function is differentiable at the indicated point. Show the work that leads to your answer.

$$(a) f(x) = \begin{cases} 5 - 4x, & x \leq 0 \\ -2x^2, & x > 0 \end{cases} \text{ at } x = 0$$

$$(b) f(x) = \begin{cases} (x-1)^3, & x \leq 1 \\ (x-1)^2, & x > 1 \end{cases} \text{ at } x = 1$$

10. **True or False. If false, explain why or give a counterexample.**

- (a) The slope of the tangent line to the differentiable function f at the point $(2, f(2))$ is

$$\frac{f(2+h) - f(2)}{h}.$$

- (b) If a function is continuous at a point, then that function is differentiable at that point.
- (c) If a function's slopes from both the right and the left at a point are the same, then that function is differentiable at that point.
- (d) If a function is differentiable at a point, then that function is continuous at that point.

11. Using your **calculator** to zoooooom in, determine if $h(x) = \sqrt{x^2 + 0.0001} + 0.99$ is locally linear at $x = 0$. Give a reason for your answer.

Multiple Choice

- _____ 12. A function will fail to be differentiable at all of the following except

- (A) A vertical asymptote (B) A removable discontinuity (C) A cusp
(D) A vertical tangent line (E) A horizontal tangent line

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ 1, & x = 2 \end{cases}$$

- _____ 13. Let f be the function defined above. Which of the following statements about f are true?

- I. $\lim_{x \rightarrow 2} f(x)$ exists
II. f is continuous at $x = 2$
III. f is differentiable at $x = 2$

- (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

- _____ 14. Let f be a differentiable function such that $f(2) = 1$ and $f'(2) = 4$. Let $T(x)$ be the equation of the tangent line to $f(x)$ at $x = 2$. What is the value of $T(1.9)$?

- (A) 0.4 (B) 0.6 (C) 0.7 (D) 1.3 (E) 1.4

_____ 15. Let f be a function such that $\lim_{h \rightarrow 0} \frac{f(7+h) - f(7)}{h} = 5$. Which of the following must be true?

- I. f is continuous at $x = 7$
- II. f is differentiable at $x = 7$
- III. The derivative of f is differentiable at $x = 7$

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

_____ 16. At $x = 4$, the function given by $h(x) = \begin{cases} x^2, & x \leq 4 \\ 4x, & x > 4 \end{cases}$ is

- (A) Undefined
- (B) Continuous but not differentiable
- (C) Differentiable but not continuous
- (D) Neither continuous nor differentiable
- (E) Both continuous and differentiable