

Name _____ Date _____ Period _____

Worksheet 2.3—Differentiation Rules

Show all work. No Calculator unless stated otherwise.

Short Answer

For 1-3, using correct notation (always), find the derivatives of the following functions. Simplify early and often!! Be sure to consider rewriting each term in the correct form first.

$$1. f(x) = -2x^3 - x^2 + 4x - 7$$

$$2. g(x) = \frac{1}{x} - 3\sin x + \frac{4x + 2\sqrt{x}}{3\sqrt[3]{x}}$$

$$3. y = 4x^2(3 - 2x)^2$$

For 4-6, using correct notation (never not always), evaluate each of the following with respect to the indicated variable. Simplify early and often!! Be sure to consider rewriting each term in the correct form first.

$$4. \frac{d}{dt} [\sqrt{3t} - 6\sqrt[4]{t} - 4\cos t - \pi] =$$

$$5. \frac{d}{dx} \left[\left(\frac{x}{x^2 + 1} \right)^{-1} \right] =$$

$$6. \frac{d}{dm} \left[\frac{m^{-1} + m^{-2}}{m^{-3}} \right] =$$

7. For $f(x) = (x^2 + 2x)(x+1)$, find $f'(x)$. Remember to simplify early and often!

(a) find the equation of the tangent line, in Taylor form, to the graph of f at $x = 1$.

(b) find the equation of the normal line, in Taylor form, to the graph of f at $x = 1$.

(c) Using your equation from part (b), find the x -intercept of the normal line. Show the work that leads to your answer.

For 8-10, determine the point(s) (if any) at which the graph of the following functions have horizontal tangent lines. Justify.

8. $y = x + \sin x$, $0 \leq x < 2\pi$

9. $y = \sqrt{3}x + 2\cos x$, $x \in [0, 2\pi)$

10. $f(x) = \frac{2}{x^2}$

For 11 & 12, find the value of k such that the given line is tangent to the graph of the given function.

11. $f(x) = x^2 - kx$, line $y = 4x - 9$

12. $f(x) = k\sqrt{x}$, line $y = x + 4$

Questions 13-16 are True or False. If False, either explain why, rewrite it to make it true, or provide a counterexample.

13. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

14. If $f(x) = g(x) + C$, then $f'(x) = g'(x)$.

15. If $y = \pi^3$, then $\frac{dy}{dx} = 3\pi^2$.

16. If $f(x) = \frac{1}{x^n}$, then $f'(x) = \frac{1}{nx^{n-1}}$

17. (Calculator permitted) A priceless Faberge egg is dropped from the top of a building that is 1362 feet tall. The egg's height in feet at time t seconds is given by $h(t) = -16t^2 + 1362$.
- (a) Find the average velocity, in ft/sec, of the egg on the interval $[1, 2]$ seconds. Show the work that leads to your answer (always).
- (b) Based on your answer in part (a), fill in the blanks so that the following sentence verbally describes the result found above.
- “On the interval from $t =$ _____ seconds to $t =$ _____ seconds, the egg's height is _____ (increasing/decreasing), on average, by _____ feet per second.”
- (c) Find the instantaneous velocity of the egg at $t = 2$ seconds. Show the work that leads to your answer (always).
- (d) Based on your answer in part (c), fill in the blanks so that the following sentence verbally describes the result found above.
- “At $t = 2$ _____, the egg's height is _____ (increasing/decreasing), by 64 _____.”
- (e) After how many seconds will the egg hit the ground? Show the work that leads to your answer.
- (f) Find the velocity, in ft/sec, of the egg as it hits the ground.

18. Find the values of a and/or b ($a \neq 0$), if they exist, such that f is differentiable for all x . As always, show the work that leads to your answer.

$$(a) f(x) = \begin{cases} ax^3, & x \leq 2 \\ x^2 + b, & x > 2 \end{cases}$$

$$(b) f(x) = \begin{cases} \cos x, & x < \frac{\pi}{2} \\ ax + b, & x \geq \frac{\pi}{2} \end{cases}$$

$$(c) f(x) = \begin{cases} \sin x, & x \leq 0 \\ ax, & x > 0 \end{cases}$$

$$(d) f(x) = \begin{cases} ax^2, & x \leq 1 \\ b\sqrt{x}, & x > 1 \end{cases}$$

Multiple Choice

_____ 19. The function $f(x) = 3\sqrt[3]{x^2} + x - 1$ is differentiable for which values of x ?

- (A) all real numbers (B) $x \in [0, \infty)$ (C) $x \in (0, \infty)$ (D) for all $x \neq 0$ (E) $(-\infty, 0)$

_____ 20. The function $f(x) = |x+4| - \sqrt[5]{x^3} + \frac{1}{x-3}$ is differentiable for all x -values except

- I. $x = -4$
II. $x = 0$
III. $x = 3$

- (A) I only (B) I and II only (C) I and III only (D) III only (E) I, II, and III

_____ 21. On the interval $[0, 2\pi)$, for which of the following x -values is the function $f(x) = \tan x$ not differentiable?

- (A) all real numbers (B) $x = \frac{\pi}{4}$ (C) $x = \frac{3\pi}{2}$ (D) $x = \pi$ (E) $x = 0$