Worksheet 3.3—Increasing, Decreasing, and 1st Derivative Test

Show all work. No calculator unless otherwise stated.

Multiple Choice

1. Determine the increasing and decreasing open intervals of the function
   \( f(x) = (x - 3)^{4/5} (x + 1)^{1/5} \) over its domain. Tip: factor out least powers from the derivative to put it into full-fledged-factored-form!

   (A) Inc: \((-1, \frac{1}{5})\), Dec: \((-\frac{1}{5}, \infty)\)
   (B) Inc: \((-1, \frac{1}{5}) \cup (3, \infty)\), Dec: \((-\frac{1}{5}, 3)\)
   (C) Inc: \((\infty, -1) \cup (3, \infty)\), Dec: \((-1, 3)\)
   (D) Inc: \((-\infty, \frac{1}{5}) \cup (3, \infty)\), Dec: \((-\frac{1}{5}, 3)\)
   (E) Inc: \((-\frac{1}{5}, 3) \cup (3, \infty)\), Dec: \((-1, \frac{1}{5}) \cup (3, \infty)\)

2. Let \( f \) be the function defined by \( f(x) = x - \cos 2x, \ -\pi \leq x \leq \pi \). Determine all open interval(s) on which \( f \) is decreasing.

   (A) \((-\frac{5\pi}{12}, \frac{\pi}{12}) \cup \frac{7\pi}{12}, \frac{11\pi}{12}\)
   (B) \((-\frac{5\pi}{12}, \frac{\pi}{6}) \cup \frac{\pi}{6}, \frac{11\pi}{12}\)
   (C) \((-\frac{5\pi}{12}, -\frac{\pi}{8}) \cup \frac{3\pi}{8}, \frac{11\pi}{12}\)
   (D) \(-\frac{\pi}{6}, -\frac{\pi}{12}) \cup \frac{\pi}{6}, \frac{11\pi}{12}\)
   (E) \(-\pi, -\frac{5\pi}{12}) \cup \frac{7\pi}{12}, \pi\)
3. Let \( f(x) = x \left( 4 + x^2 - \frac{x^4}{5} \right) \).

____ (i) Which of the following is \( f'(x) \)?

(A) \( f''(x) = (1 + x^2)(5 - x^2) \)

(B) \( f''(x) = (1 + x^2)(4 - x^2) \)

(C) \( f''(x) = (1 - x^2)(5 + x^2) \)

(D) \( f''(x) = (1 - x^2)(4 + x^2) \)

(E) \( f''(x) = (1 - x^2)(4 - x^2) \)

____ (ii) Find the open interval(s) on which \( f \) is increasing.

(A) \((-\infty, -2) \cup (2, \infty)\)

(B) \((-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)\)

(C) \((-2, 2)\)

(D) \((-\infty, -1) \cup (1, \infty)\)

(E) \((-1, 1)\)

4. The derivative of a function \( f \) is given for all \( x \) by \( f'(x) = (2x^2 + 4x - 16)(1 + g^2(x)) \) where \( g \) is some unspecified function. At which value(s) of \( x \) will \( f \) have a local maximum?

Note: \( g^2(x) = (g(x))^2 \)

(A) \( x = -4 \)

(B) \( x = 4 \)

(C) \( x = -2 \)

(D) \( x = 2 \)

(E) \( x = -4, 2 \)
5. Which of the following statements about the absolute maximum and absolute minimum values of
\[ f(x) = \frac{x^3 - 4x^2 - 6x - 1}{x + 1} \]
on the interval \([0, \infty)\) are correct? (Hint: Think of what type of discontinuity does \( f(x) \) have???)

0 or \( \frac{0}{0} \)

(A) Max = 13, No Min

(B) No Max, Min = \( -\frac{29}{4} \)

(C) Max = 13, Min = \( -\frac{29}{4} \)

(D) Max = 5, No Min

(E) No Max, Min = 1

6. (Calculator Permitted) The first derivative of the function \( f \) is defined by \( f'(x) = \cos(x^3 - x) \) for \( 0 \leq x \leq 2 \). On what intervals is \( f \) increasing?

(A) \( 0 \leq x \leq 1.445 \) only

(B) \( 1.445 \leq x \leq 1.875 \)

(C) \( 1.691 \leq x \leq 2 \)

(D) \( 0 \leq x \leq 1 \) and \( 1.691 \leq x \leq 2 \)

(E) \( 0 \leq x \leq 1.445 \) and \( 1.875 \leq x \leq 2 \)
Short Answer

7. For each of the following, find the critical values (on the indicated intervals, if indicated.) Remember, a critical value MUST be in the domain of the function, though it may not be in the domain of the derivative.

   (a) \( f(x) = x^2 (3 - x) \)  
   (b) \( f(x) = \frac{\sin x}{1 + \cos^2 x}, \ [0, 2\pi] \)  
   (c) \( f(x) = \frac{x^2}{x^2 - 9} \)

8. Determine the local extrema of each of the following functions (on the indicated interval, if indicated). Be sure to say which type each is. **Justify** (this means write a sentence.)

   (a) \( f(x) = \cos^2 (2x), \ [0, 2\pi] \)  
   (b) \( f(x) = x + \frac{1}{x} \)  
   (c) \( f(x) = \sin^2 x + \sin x, \ [0, 2\pi] \)  
   (d) \( f(x) = \frac{x^2 - 3x - 4}{x - 2} \)
9. Assume that \( f \) is differentiable for all \( x \). The signs of \( f' \) are as follows.

\[
    f'(x) > 0 \text{ on } (-\infty,-4) \cup (6,\infty) \text{ and } f'(x) < 0 \text{ on } (-4,6)
\]

Let \( g(x) \) be a transformation of \( f(x) \). Supply the appropriate inequality (\( >, <, \geq, \leq \)) for the indicated value of \( c \) in the given blank.

<table>
<thead>
<tr>
<th>Function</th>
<th>Sign of ( g'(c) )</th>
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</thead>
<tbody>
<tr>
<td>( g(x) = f(x) + 5 )</td>
<td>( g'(0) ) 0</td>
</tr>
<tr>
<td>( g(x) = 3f(x) - 3 )</td>
<td>( g'(-5) ) 0</td>
</tr>
<tr>
<td>( g(x) = -f(x) )</td>
<td>( g'(-6) ) 0</td>
</tr>
<tr>
<td>( g(x) = -f(x) )</td>
<td>( g'(0) ) 0</td>
</tr>
<tr>
<td>( g(x) = f(x-10) )</td>
<td>( g'(0) ) 0</td>
</tr>
<tr>
<td>( g(x) = f(x-10) )</td>
<td>( g'(8) ) 0</td>
</tr>
<tr>
<td>( g(x) = f(-x) )</td>
<td>( g'(8) ) 0</td>
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<tr>
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<td>( g'(-8) ) 0</td>
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