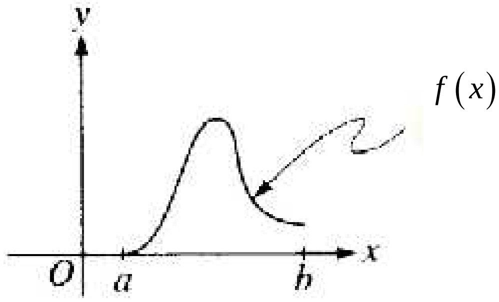
Name____ Date_______________________ Period______

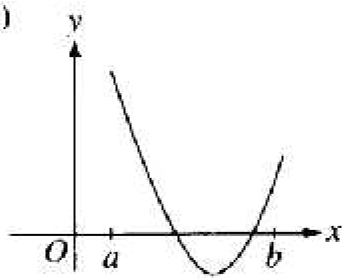
Worksheet 3.5— f, f', f''

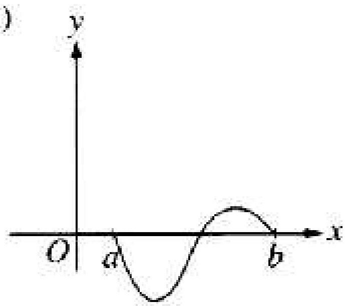
Show all work. No calculator unless otherwise stated.

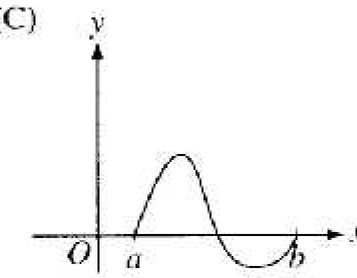
Multiple Choice

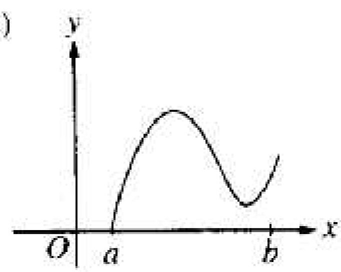


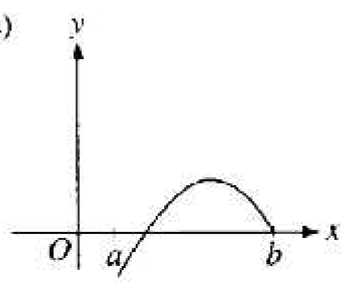
_____ 1. The graph above shows the graph of $f(x)$ for some function $f(x)$ on $[a,b]$. Which of the following graphs could be the graph of $f'(x)$ on $[a,b]$?

(A)


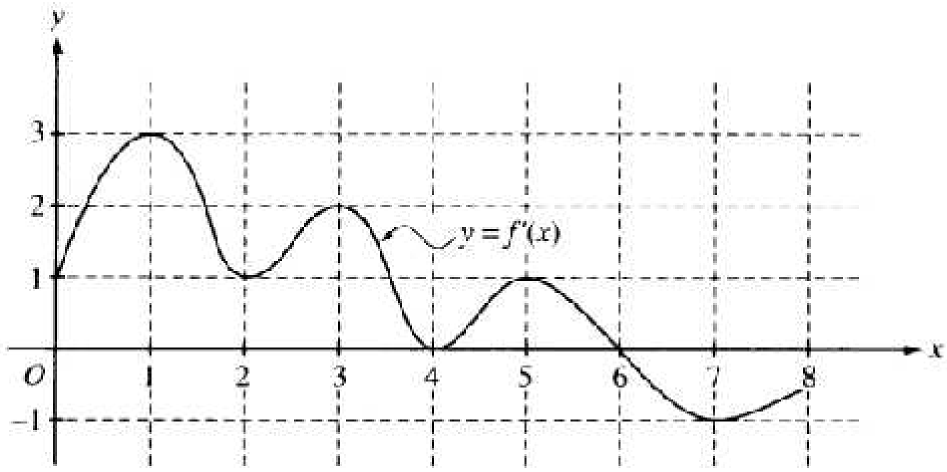
(B)


(C)


(D)


(E)


<u>Questions 2 - 3</u> refer to the graph and information below.



The graph above shows $f'(x)$ for some function $f(x)$ on $[0,8]$.

_____ 2. How many points of inflection does the graph of f have on $[0,8]$?

(A) Two

(B) Three

(C) Four

(D) Five

(E) Six

_____ 3. Which of the following accurately describes the relative extrema of $f(x)$ on $[0,8]$?

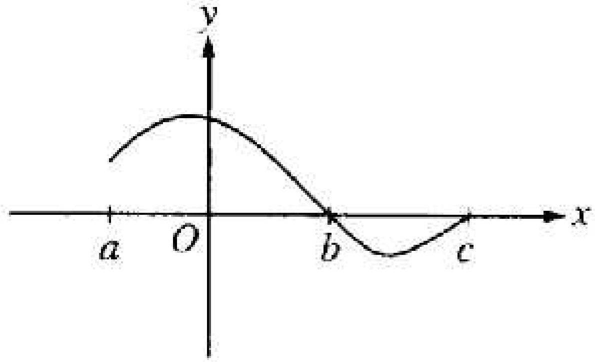
(A) 3 Relative Maxima and 3 Relative Minima

(B) 2 Relative Maxima and no Relative Minima

(C) 1 Relative Maximum and 1 Relative Minimum

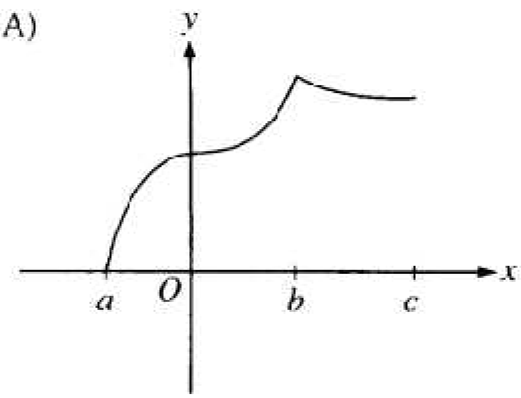
(D) 1 Relative Maximum and no Relative Minima

(E) No Relative Maxima and 1 Relative Minimum

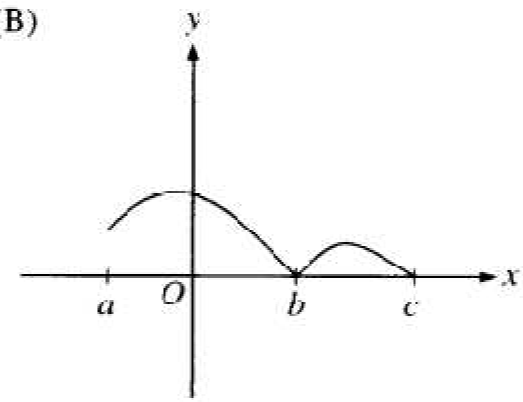


_____ 4. The graph above shows the graph of $f'(x)$ for some function $f(x)$ on $[a,c]$. Which of the following could be the graph of $f(x)$?

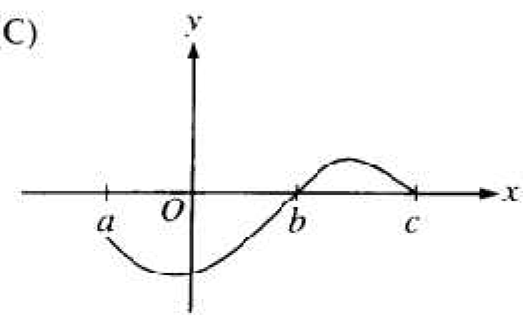
(A)



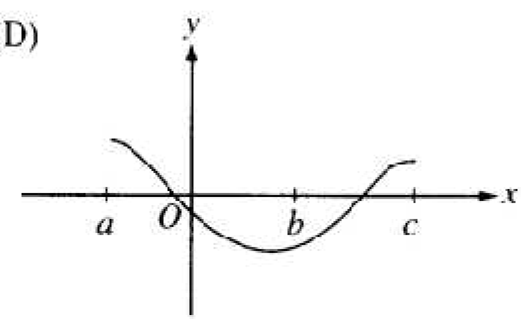
(B)



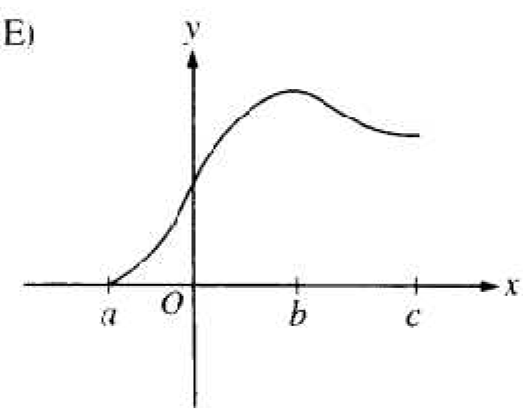
(C)



(D)

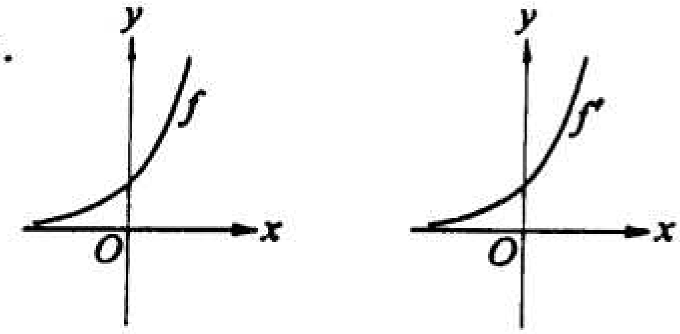


(E)

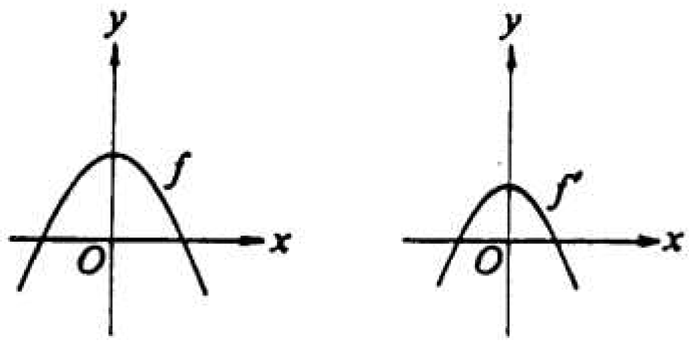


_____ 5. Which of the following could represent the graph of a function $f(x)$ and its derivative $f'(x)$?

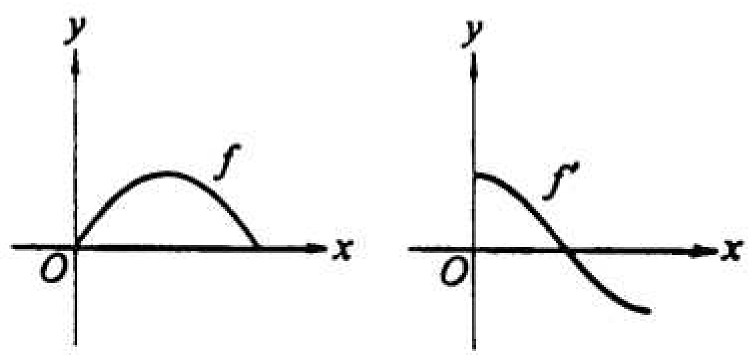
I.



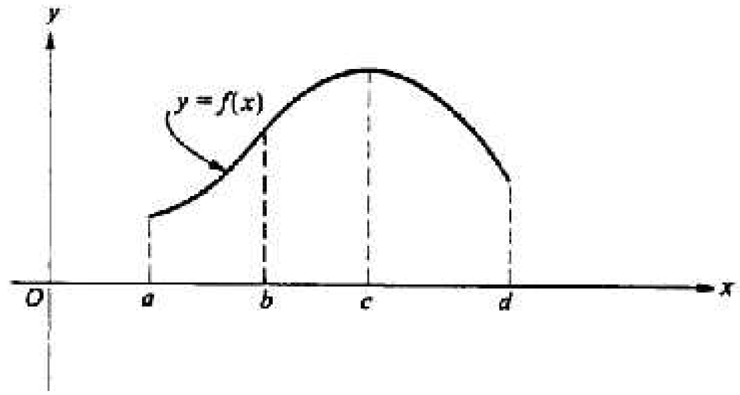
II.



III.



(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

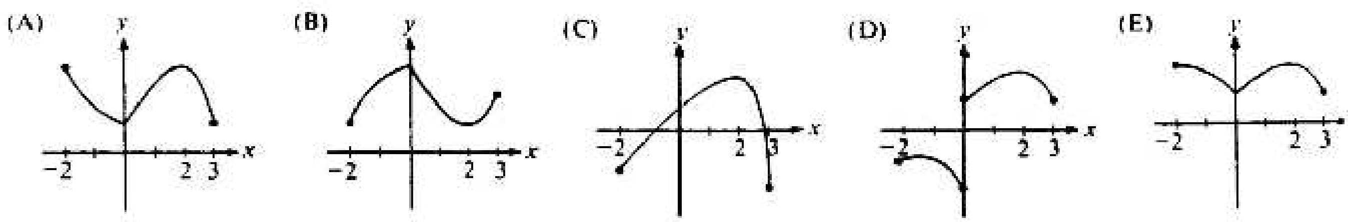


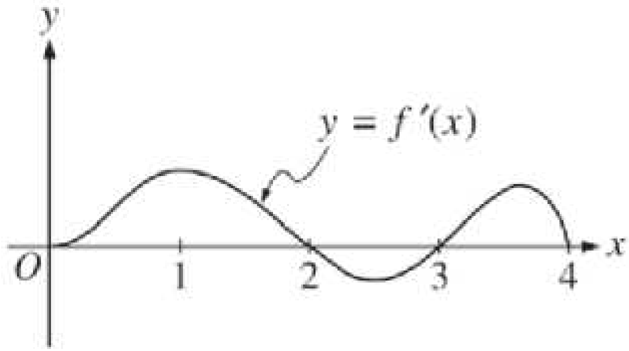
_____ 6. The graph of $y = f(x)$ is shown in the figure above. If f has a critical value at $x = c$ and an inflection value at $x = b$, on which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II only (E) II and III only

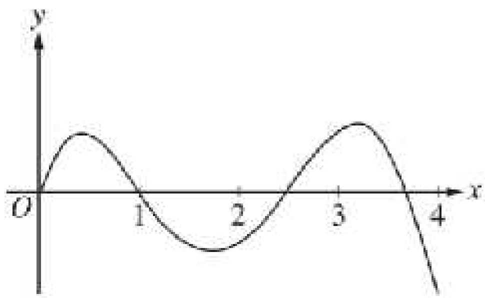
_____ 7. Let f be a function that is continuous on the closed interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f.



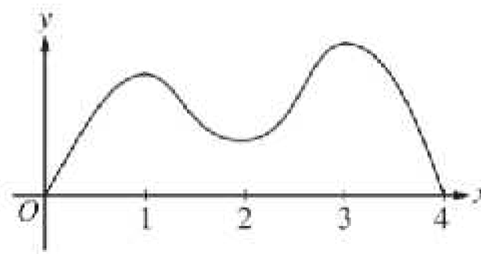


_____ 8. The figure above shows the graph of f', the derivative of the function f. If $f(0)=0$, which of the following could be the graph of f?

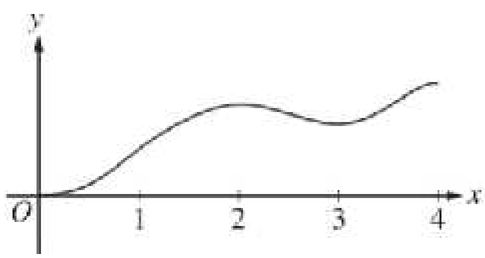
(A)



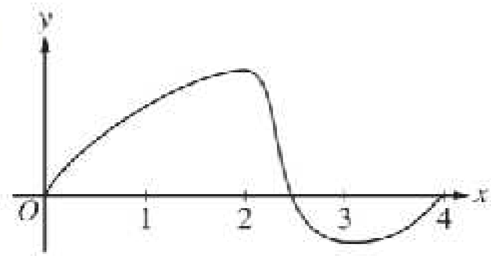
(B)



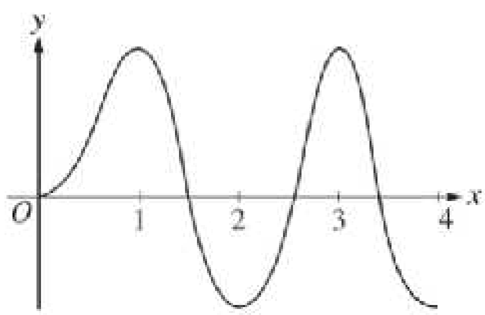
(C)



(D)



(E)



Short Answer

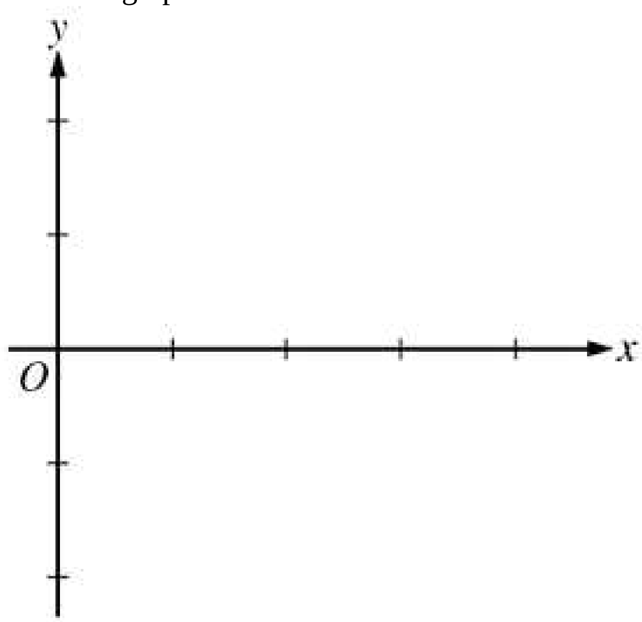
9. (AB4 2005) Let *f* be a function that is continuous on the interval $[0,4)$. The function *f* is twice differentiable except at $x = 2$. The function *f* and its derivatives have the properties indicated in the table below.

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$
$f(x)$	–1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	–3	Negative
$f''(x)$	–2	Negative	0	Positive	DNE	Negative	0	Positive

(a) For $0 < x < 4$, find all values of *x* at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) Find the coordinates of any inflection points on the graph of *f*. Justify your answer.

(c) On the axes below, sketch the graph of a function that has all the characteristics of *f*.



10. (AB3 1981) Let f be the function defined by $f(x) = 12x^{2/3} - 4x$

(a) Find the intervals on which f is increasing.

(b) Find the x- and y-coordinates of all relative maximum points. Justify.

(c) Find the x- and y-coordinates of all relative minimum points. Justify.

(d) Find the intervals on which f is concave downward.

(e) Using the information found in parts (a), (b), (c), and (d), sketch the graph of f.

11. (AB5 1980) Given the function f defined by $f(x) = \cos x - \cos^2 x$ for –® ≤ x ≤ ® .

(a) Find the x-intercepts of the graph of f.

(b) Find the x- and y-coordinates of all relative maximum points. Justify.

(c) Find the intervals on which the graph of f is increasing.

(d) Using the information found in parts (a), (b), and (c), sketch the graph of f.

12. Sketch the graph of a function that satisfies all of the following conditions.

(a) $f'(x) > 0$ for all $x \neq 1$, $\lim_{x \to 1^-} f(x) = \infty$, $\lim_{x \to 1^+} f(x) = -\infty$, $f''(x) > 0$ if $x < 1$ or $x > 3$, and $f''(x) < 0$ if $1 < x < 3$.

(b) $f'(x) > 0$ if $-2 < x < 2$, $f'(x) < 0$ if $x < -2$ and $x > 2$, $f'(2) = 0$, $\lim_{x \to \infty} f(x) = 1$, $f(-x) = -f(x)$, $f''(x) < 0$ if $0 < x < 3$, and $f''(x) > 0$ if $x > 3$

For 13 - 15, use your knowledge of f, f', f'' along with any other non-calculator information you can gather (intercepts, end-behavior, discontinuities, symmetry, etc) to sketch the graphs of the following function.

13. $f(x) = 2x^{5/3} - 5x^{4/3}$

.

14. $g(x) = x\sqrt[3]{x^2 - 4}$

15. $h(x) = \dfrac{x^3}{x^2 + 1}$