Name_____ Date_____ Period_____

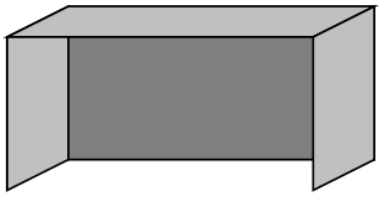
Worksheet 3.6—Optimization
Show all work. Calculator permitted. Show all set-ups and analysis. Report all answers to 3 decimals and avoid intermediate rounding error.

Multiple Choice

1. An advertisement is run to stimulate the sale of cars. After t days, $1 \le t \le 48$, the number of cars sold is given by $N(t) = 4000 + 45t^2 - t^3$. On what day does the maximum **rate of growth** sales occur?

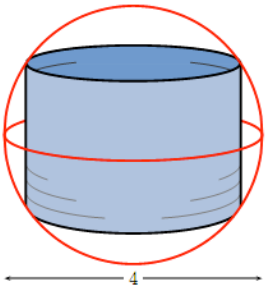
(A) on day 17 (B) on day 13 (C) on day 15 (D) on day 16 (E) on day 14

2. A canvas wind shelter like the one at right is to be built for use along parts of the Guadalupe River. It is to have a back, two square sides, and a top. If $\frac{147}{2}$ square feet of canvas is to be used in the construction, find the depth of the shelter for which the space inside is maximized assuming all the canvas is used.



(A) depth $= \frac{7}{2}$ feet (B) depth $= \frac{7}{4}$ feet (C) depth $= 4$ feet (D) depth $= 7$ feet (E) none of these

3. A right circular cylinder is inscribed in a sphere with diameter 4cm as shown. If the cylinder is open at both ends, find the largest possible surface area of the cylinder.



(A) $A = 8 \text{ cm}^2$ (B) $A = 16 \text{ cm}^2$ (C) $A = 16\pi \text{ cm}^2$

(D) $A = 2 \text{ cm}^2$ (E) $A = 8\pi \text{ cm}^2$ (F) $A = 4\pi \text{ cm}^2$

(G) $A = 4 \text{ cm}^2$ (H) $A = 2\pi \text{ cm}^2$ (I) $A = 2 \text{ cm}^2$

4. The point on the curve $y = \sqrt{2x}$ that is nearest the point $(1,4)$ occurs at the point

(A) $\left(1,\sqrt{2}\right)$ (B) $(2,2)$ (C) $\left(4,2\sqrt{2}\right)$ (D) $(0,0)$ (E) $(8,4)$

5. A point moves on the *x*-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $\sqrt{e}$ (C) e (D) $\sqrt{e^3}$ (E) There is no maximum value for v.

6. The *derivative* of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

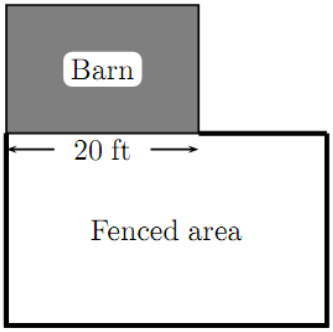
(A) -1 (B) 0 (C) 1 (D) $\frac{4}{3}$ $\frac{5}{3}$

7. If $y = 2x - 8$, what is the minimum value of the product xy?

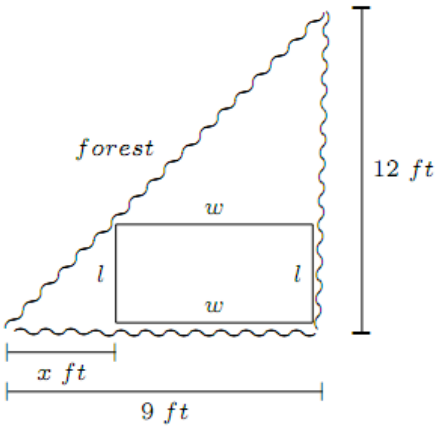
(A) -16 (B) -8 (C) -4 (D) 0 (E) 2

Short Answer:

8. A farmer wishes to erect a fence enclosing a rectangular area adjacent to a barn which is 20 feet long. The diagram illustrates his plan for the fenced area. Find the largest area that can be enclosed if 96 feet of fencing material is available. Justify your answer.



9. Suppose the same farmer from Question 8 has to build a rectangular pig pen on a triangular-shaped piece of land bounded by a 12 foot stretch of river, a 9 foot stretch of wall, and a 15 foot stretch of forest (shown at right). Find the dimensions of the rectangular pen that he can build that maximizes the area enclosed by the fence.



10. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10 per square foot and the material used to build the sides cost $6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.

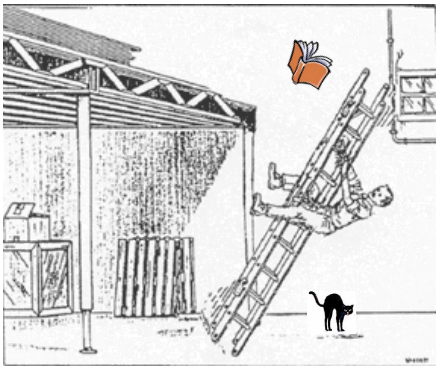
11. Mr. Wenzel has a big problem. He has a sphere of radius 3 feet ands he is trying to find the volume of a right circular cylinder with maximum volume that can be inscribed inside his sphere. Aside from any problems of actually getting the cylinder into the sphere, help Mr. Wenzel find the volume of the aforementioned right circular cylinder.

12. A rectangle has its vertices on the *x*-axis, the *y*-axis, the origin, and the graph of $y = 4 - x^2$ in the first quadrant. Find the maximum possible area for such a rectangle. Justify your answer

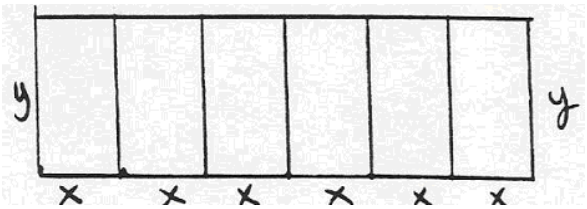
13. Farmer Tate's apple orchard now has 30 trees per acre, and the average yield is 400 apples per tree. For each additional tree that he plants per acre, the average yield per tree is reduced by approximately 10 apples. How many trees per acre will give Farmer Tate the largest crop of apples?

14. A Norman window has a lower section in the shape of a rectangle and an upper portion in the shape of a semicircle mounted on the upper side of this rectangle. If the window is to be surrounded by 10 feet of metal border (not including the part connecting the rectangle to the semi-circle), find the ratio of the overall height to the overall width if the total area of the window is to be a maximum.

15. A ladder (with Korpi on the rungs) is to reach over a fence 8 feet high to a wall one foot behind the fence. This will allow Korpi access to a window where a dastardly student awaits to drop a calculus book on his head. What is the length of the shortest ladder that can be used? Assume a black cat will be beneath the ladder.



16. Jenna wants to build a motel for her pet ladybugs. She envisions a six-room motel to be built with the floor plan shown. Each room is to have 350 square inches of floor space, bits of cracker crumbs, a whirlpool ladybug spa, a lamp with a burned-out bulb, and a picture of Hannah Montana on the wall. What dimensions should the rooms be in order to have the minimum total length of walls to build? All rooms are identical in size and have walls painted Red. You may use your calculator to graph the derivative.



17. Find the point on the parabola $y = 9 - x^2$ closest to the point $(3,9)$. Justify your answer.

18. Find a positive number such that the sum of the number and its reciprocal is as small as possible.

19. Find the positive number such that the sum of 6 times this number and 4 times its reciprocal is as small as possible.

20. Calculator permitted: A storage bin is to be constructed by removing a sector with a central angle, θ, from a circular piece of tin of radius 5ft and folding the remainder of tin to form a cone. What should θ be in order to obtain a maximum volume of a storage bin formed in this fashion? (Hint: Find an equation for Volume in terms of θ in radians. Plot this function on a reasonable window and find the maximum numerically.) Box your final Volume equation as a function of θ, write down your graphing window (x and y), and give your final answer in radians and degrees to three decimal places. Did you get the same answer as Farmer Leibniz? If not, start over, this time more correctly. You will need the arc length, s, formula: $s = r\theta$, where θ is in radians.