Worksheet 4.3—The Fundamental Theorem of Calculus
Show all work. No calculator unless otherwise stated.

**Multiple Choice**

1. (Calculator Permitted) What is the average value of \( f(x) = \cos x \) on the interval \([1,5]\)?
   - (A) –0.990
   - (B) –0.450
   - (C) –0.128
   - (D) 0.412
   - (E) 0.998

2. If the average value of the function \( f \) on the interval \([a,b]\) is 10, then \( \int_a^b f(x)dx = \)
   - (A) \( \frac{10}{b-a} \)
   - (B) \( \frac{f(a) + f(b)}{10} \)
   - (C) \( 10b - 10a \)
   - (D) \( \frac{b-a}{10} \)
   - (E) \( \frac{f(a) + f(b)}{20} \)

3. (Calculator Permitted) Let \( f'(x) = \ln(2 + \sin x) \). If \( f(3) = 4 \), then \( f(5) = \)
   - (A) 0.040
   - (B) 0.272
   - (C) 0.961
   - (D) 4.555
   - (E) 6.667
4. What is \( \lim_{h \to 0} \frac{1}{h} \int_{x}^{x+h} f(t) \, dt \) ?

(A) 0  (B) 1  (C) \( f'(x) \)  (D) \( f(x) \)  (E) nonexistent

5. What is the linearization of \( f(x) = \int_{x}^{\pi} \cos^3 t \, dt \) at \( x = \pi \) ?

(A) \( y = -1 \)  (B) \( y = -x \)  (C) \( y = \pi \)  (D) \( y = x - \pi \)  (E) \( y = \pi - x \)

6. (Calculator Permitted) The area of the region enclosed between the graph of \( y = \sqrt{1-x^4} \) and the x-axis is

(A) 0.886  (B) 1.253  (C) 1.414  (D) 1.571  (E) 1.748
Short Answer

7. Let \( f \) be a function such that \( f''(x) = 6x + 12 \).
   (a) Find \( f(x) \) if the graph of \( f \) is tangent to the line \( 4x - y = 5 \) at the point \((0, -5)\)

   (b) Find the average value of \( f(x) \) on the closed interval \([-1, 1]\).

8. Suppose \( f \) has a negative derivative for all values of \( x \) and that \( f(1) = 0 \). Which of the following statements must be true of the function

   \[ h(x) = \int_{0}^{x} f(t) \, dt \]

   Give reasons for your answers.
   (a) \( h \) is a twice-differentiable function of \( x \).
   (b) \( h \) and \( dh/dx \) are both continuous.
   (c) The graph of \( h \) has a horizontal tangent at \( x = 1 \).
   (d) \( h \) has a local maximum at \( x = 1 \).
   (e) \( h \) has a local minimum at \( x = 1 \).
   (f) The graph of \( h \) has an inflection point at \( x = 1 \).
   (g) The graph of \( dh/dx \) crosses the \( x \)-axis at \( x = 1 \).
9. Find \( \frac{dy}{dx} \)

(a) \( y = \int_{-\pi}^{x} \frac{2 \sin t}{3 + \cos t} \, dt \)

(b) \( y = \int_{x}^{7} \sqrt{2m^4 + m + 1} \, dm \)

(c) \( y = \int_{x}^{5} \frac{\cos t}{t^2 + 1} \, dt \)

(d) \( y = \int_{\sqrt{x}}^{x} \sqrt{u} \sin u \, du \)

10. If \( F(x) = \int_{1}^{x} f(t) \, dt \), where \( f(t) = \int_{1}^{t} \sqrt{1 + u^4} \, du \), find \( F''(2) \).
11. (Calculator Permitted) If \( \frac{dy}{dx} = \sin^3 x \) and \( y = 4 \) when \( x = 5 \), construct and evaluate an integral equation to find

(a) \( y(7) \)  
(b) \( y(0) \)  
(c) \( y(-2) \)  
(d) \( y(x) \)

12. Evaluate without a calculator, then verify using fnINT(

(a) \( \int_{-1}^{2} 3^x \, dx \)  
(b) \( \int_{-2}^{x^2} \frac{1}{x^2} \, dx \)  
(c) \( \int_{0}^{x^2 + \sqrt{x}} \, dx \)  
(d) \( \int_{\pi/6}^{5\pi/6} \csc^2 \theta \, d\theta \)  
(e) \( \int_{0}^{4} \frac{1 - \sqrt{u}}{\sqrt{u}} \, du \)

(f) \( \int_{0}^{2} x(2 + x^5) \, dx \)  
(g) \( \int_{0}^{\frac{4}{t^2 + 1}} \, dt \)  
(h) \( \int_{0}^{2} f(x) \, dx \) where \( f(x) = \begin{cases} x^4, & 0 \leq x < 1 \\ x^5, & 1 \leq x \leq 2 \end{cases} \)
13. Find the area of the region bounded by the $x$-axis and the curve $y = x^3 - 4x$ on $-2 \leq x \leq 2$

14. If $f(1) = 12$, $f'(x)$ is continuous, and $\int_{1}^{4} f'(x) = 17$, what is the value of $f(4)$?

15. Find the average value of the following function on the given intervals. Verify with fnINT(
   (a) $f(x) = \cos x$ on $[0, \pi/2]$  
   (b) $f(x) = 1/x$ on $[1, 4]$  
   (c) $y = \sec x \tan x$ on $[0, \pi/4]$
16. The graph of $f$ is shown above. If $F(x) = \int_2^x f(t) \, dt$, evaluate the following using areas to help you.

(a) $F(0)$  
(b) $F(2)$  
(c) $F(5)$  
(d) $F(7) - F(5)$  
(e) $F(9)$

(f) where does $F$ have a maximum value? A minimum value?

(g) What is the average value of $f(x)$ on $[2, 9]$?
17. Let \( g(x) = \int_{0}^{x} f(t) \, dt \), where \( f \) is the function whose graph is given below.

(a) At what values of \( x \) do the local maximum and local minimum of \( g \) occur? Justify.

(b) Where does \( g \) attain its absolute maximum value?

(c) On what approximate intervals is \( g \) concave downward?

(d) Sketch the graph of \( g \).
18. (Calculator Permitted) If a cup of coffee has temperature $95^\circ C$ in a room where the temperature is $20^\circ C$, then, according to Newton’s Law of Cooling, the temperature of the coffee after $t$ minutes is $T(t) = 20 + 75e^{-t/50}$.

What is the average temperature of the coffee during the first half hour? Show your integral set up. Include units in your final answer.