

Name _____ Date _____ Period _____

Worksheet 4.4—Integration by u -Substitution and Pattern RecognitionShow all work. **No calculator unless otherwise stated.**

Multiple Choice:

1. Find the most general function f such that $f''(x) = 9 \cos 3x$

(A) $f(x) = -3 \sin x + Cx^2 + D$ (B) $f(x) = -\cos 3x + Cx + D$ (C) $f(x) = -3 \cos 3x + Cx^2 + D$

(D) $f(x) = \sin x + Cx + D$ (E) $f(x) = 3 \sin 3x + Cx + D$

2. Evaluate the definite integral: $\int_0^1 (1 + e^{-x})^2 dx$.

(A) $\frac{3}{2} - 2e + \frac{1}{2}e^2$ (B) $\frac{7}{2} + \frac{2}{e} + \frac{1}{2e^2}$ (C) $\frac{3}{2} - 2e - \frac{1}{2}e^2$ (D) $\frac{3}{2} + 2e + \frac{1}{2}e^2$

(E) $\frac{7}{2} - \frac{2}{e} - \frac{1}{2e^2}$

3. Find the value of $f(-1)$ when $f'(x) = 6xe^{-2x^2}$, $f(0) = 1$.

(A) $\frac{5}{2} - \frac{3}{2}e^{-2}$ (B) $-\frac{3}{2}e^2$ (C) $-\frac{3}{2}e^{-2}$ (D) $\frac{5}{2} - \frac{3}{2}e^2$ (E) $\frac{5}{2} + \frac{3}{2}e^{-2}$

4. Evaluate the definite integral $\int_0^1 (4 - 2x)e^{8x - 2x^2} dx$

- (A) $\frac{1}{2}(e^6 - 1)$ (B) $\frac{1}{2}(e^{-6} - 1)$ (C) $e^{-6} + 1$ (D) $\frac{1}{2}(e^6 + 1)$ (E) $e^6 - 1$

5. Evaluate $\int_0^{\frac{\pi}{4}} \frac{2e^{\tan x} + 5}{\cos^2 x} dx$

- (A) $2e + 3$ (B) $2e$ (C) $2e - 3$ (D) e (E) $e + 5$

6. $\int \frac{4}{x}(1 + 2\ln x)^3 dx =$

- (A) $(1 + 2\ln x)^4 + C$ (B) $\frac{1}{2}(1 + 2\ln x)^4 + C$ (C) $-\frac{1}{2}(1 + 2\ln x)^4 + C$ (D) $\frac{1}{2}\ln x(1 + 2\ln x)^4 + C$
(E) $-(1 + 2\ln x)^4 + C$

7. Evaluate $\int_1^e \frac{1}{x} (f'(\ln x) + 2) dx$ when $f(0) = 1$ and $f(1) = 4$.

- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2

8. Evaluate $\int_0^1 \frac{6x}{1+x^2} dx$

- (A) $\frac{3}{2}$ (B) 3 (C) 6 (D) $3\ln 2$ (E) $\frac{3}{2}\ln 2$

9. Evaluate $\int_{\pi/4}^{3\pi/4} \frac{6\cos x - 2\sin x}{6\sin x + 2\cos x} dx$

- (A) $-\ln\left(\frac{5}{2}\right)$ (B) $-\ln 2$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln 2$ (E) none of these

10. Evaluate $\int_0^1 \frac{x^2 + 4x + 1}{3x^2 + 3} dx$

- (A) $\frac{1 + 4\ln 3}{6}$ (B) $\frac{1 + 2\ln 2}{3}$ (C) $\frac{2 + 4\ln 3}{3}$ (D) $\frac{1 + 2\ln 3}{3}$ (E) $\frac{1 + 4\ln 2}{6}$

11. Evaluate $\int_e^{e^4} \frac{5}{x\sqrt{\ln x}} dx$

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Free Response:

12. Evaluate the following indefinite integrals. Don't forget your $+C$.

- (a) $\int 2x(x^2 + 1) dx$ (b) $\int \frac{3t^2}{t^3 - 4} dt$ (c) $\int x\sqrt{2x^2 - 1} dx$ (d) $\int 3xe^{x^2+2} dx$

(e)
$$\int \frac{4x}{(x^2 - 8)^3} dx$$

(f)
$$\int 2re^{3r^2} dr$$

(g)
$$\int 5l^2 (l^3 - 1) dl$$

(h)
$$\int (3x^2 + 2)\sqrt{x^3 + 2x} dx$$

(i)
$$\int (6t^2 + 10t^4)(t^3 + t^5)^{100} dt$$

(j)
$$\int \frac{\ln^3 3x}{3x} dx$$

(k)
$$\int \frac{6x + 5}{3x^2 + 5x - 2} dx$$

(l)
$$\int \frac{12x + 10}{9x^2 + 15x - 6} dx$$

(m)
$$\int \frac{\cos 3x}{5 + 2 \sin 3x} dx$$

(n)
$$\int (2t + 1)e^{5t^2 + 5t} dt$$

(o)
$$\int \frac{\sin(\ln ax)}{x} dx, \text{ where } a > 0$$

(p)
$$\int \cos^3 t dt$$

13. Evaluate the following definite integrals without a calculator.

$$(a) \int_0^1 x^3 (1+x^4)^5 dx$$

$$(b) \int_{\sqrt{\pi/4}}^{\sqrt{2\pi/3}} x \sin(x^2) dx$$

$$(c) \int_{-1}^3 \sqrt{7+3x} dx$$

$$(d) \int_0^3 x\sqrt{1+x} dx$$

$$(e) \int_0^{\pi} \cos^2\left(\frac{\theta}{5}\right) \sin\left(\frac{\theta}{5}\right) d\theta$$

$$(f) \int_0^1 \frac{1+e^{3x}}{e^{3x}+3x} dx$$

$$(g) \int_0^1 \frac{1}{1+9x^2} dx$$

14. Use u -substitution to answer the following:

$$(a) \text{ If } \int \frac{dx}{1+\cos x} = \tan\left(\frac{x}{2}\right) + C, \text{ find } \int \frac{1}{3+3\cos\left(\frac{x}{4}\right)} dx \text{ exactly where } C \text{ is a constant.}$$

$$(b) \text{ If } \int_0^{\infty} \frac{x}{e^x+1} dx = \frac{\pi^2}{12}, \text{ find } \int_0^{\infty} \frac{x}{e^{5x}+1} dx \text{ exactly.}$$

15. If $\int_0^{\pi} \ln(a + b \cos(x)) dx = 2\pi \ln(2)$, for some positive a and b , find the exact value of

$$\int_0^{\frac{\pi}{5}} \ln(a + b \cos(5x)) dx .$$

16. If $\int_a^b f(x) dx = K$, evaluate the following integrals in terms of K using your knowledge of transformations.

$$(a) \int_{a+5}^{b+5} f(x-5) dx =$$

$$(b) \int_a^b [f(x) + 5] dx =$$

$$(c) \int_{\frac{a}{5}}^{\frac{b}{5}} f(5x) dx =$$

17. If $\int_3^6 f(z) dz = 4$, evaluate the following integrals exactly by using appropriate substitution and limits.

$$(a) \int_1^2 f(3z) dz$$

$$(b) \int_{0.5}^2 f(7-2z) dz$$

$$(c) \int_4^7 (f(z-1) + 5) dz$$