Name____ Date________________________ Period______

Worksheet 5.1—Separable Differential Equations
Show all work. No Calculator unless specified.

Multiple Choice

1. (Calculator Permitted) A sample of Kk-1234 (an isotope of Kulmakorpium) loses 99% of its radioactive matter in 199 hours. What is the half-life of Kk-1234?

   (A) 4 hours (B) 6 hours (C) 30 hours (D) 100.5 hours (E) 143 hours

2. In which of the following models is $\frac{dy}{dt}$ directly proportional to *y*?

   I. $y = e^{kt} + C$

   II. $y = Ce^{kt}$

   III. $y = 28^{kt}$

   IV. $y = 3\left(\frac{1}{2}\right)^{3t+1}$

   (A) I only (B) II only (C) I and II only (D) II and III only (E) II, III, and IV (F) all of them

3. (**Use your calculator on this one, but get the exact answer first.**) The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time *t*. If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?

   (A) 3.750 (B) 4.000 (C) 4.066 (D) 4.132 (E) 4.600

4. (‘88-BC43) Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

   (A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

5. ('93-AB42) (**Calculator permitted**) A puppy weighs 2.0 pounds and birth and 3.5 pounds two months later. If the weight of the puppy during the first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

6. ('98-AB84) (**Calculator permitted**) Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

7. ('93-BC38) (**Calculator Permitted**) During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

8. ('85-BC33) If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{1}{2}\ln 2$ (B) $-\frac{1}{4}$ (C) $\frac{1}{2}\ln 2$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

Free Response

For problems 9 – 18, find the general solution to the following differential equations, then find the particular solution using the initial condition. **<u>You may have to factor and/or rewrite the expression in order to separate your *x*-factors and *y*-factors.</u>**

9. $\frac{dy}{dx} = \frac{x}{y},\ y(1) = -2$

10. $\frac{dy}{dx} = -\frac{x}{y},\ y(4) = 3$

11. $\frac{dy}{dx} = \frac{y}{x},\ y(2) = 2$

12. $\frac{dy}{dx} = 2xy,\ y(0) = -3$

13. $\frac{dy}{dx} = xy + 5x + 2y + 10,\ y(0) = -1$

14. $\frac{dy}{dx} = \cos^2 y,\ y(0) = 0$

15. $\frac{dy}{dx} = -2xy^2,\ y(1) = 0.25$

16. $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x},\ y(e) = 1$

*17. $\frac{dy}{dx} = e^{x-y},\ y(0) = 2$

*18. $(\sec x)\frac{dy}{dx} = e^{y+\sin x},\ y(0) = 0$

*Xtra Spicy (requires manipulation prior to separation like **Example 4** from Notes)