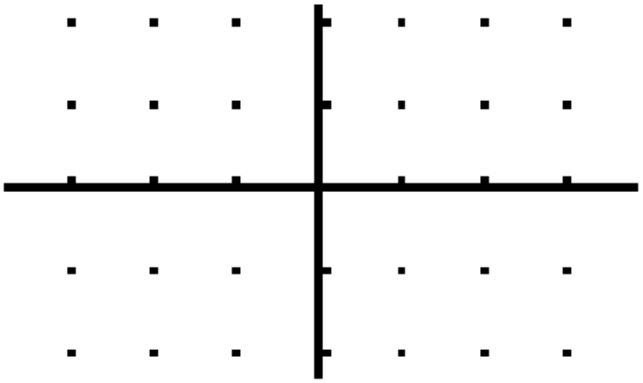
Name_____ Date_____ Period_____

Worksheet 5.2—Slope Fields
Show all work when applicable.

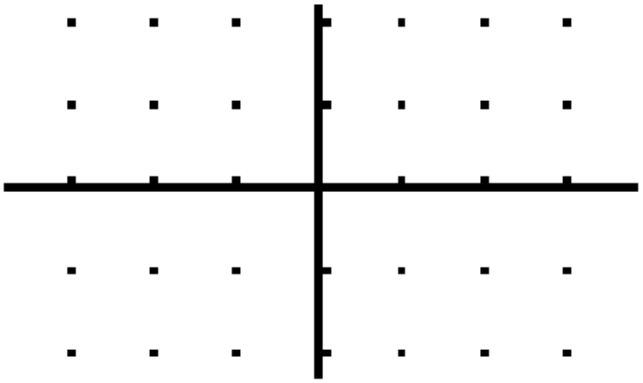
Short Answer and Free Response:

Draw a slope field for each of the following differential equations.

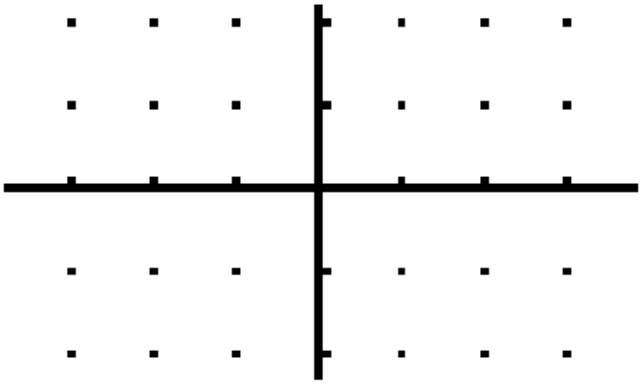
1. $\frac{dy}{dx} = x + 1$



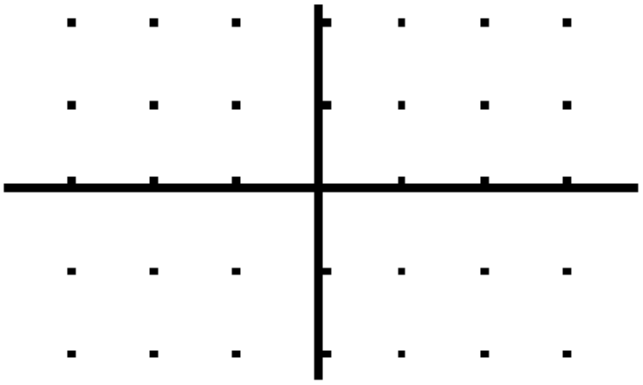
2. $\frac{dy}{dx} = 2y$



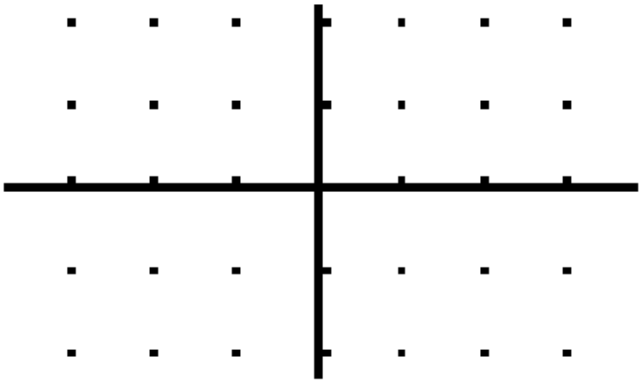
3. $\frac{dy}{dx} = x + y$



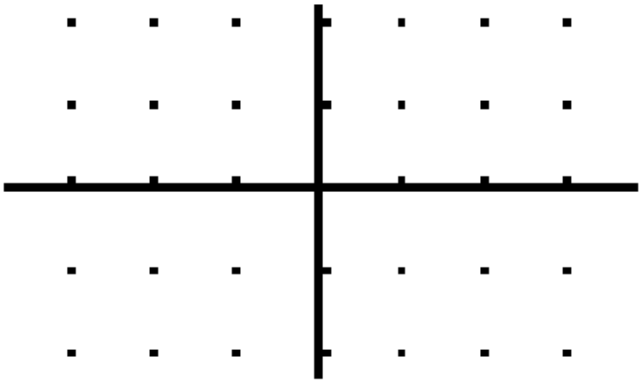
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y - 1$

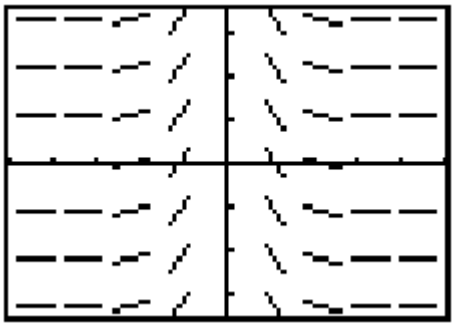


6. $\frac{dy}{dx} = -\frac{y}{x}$

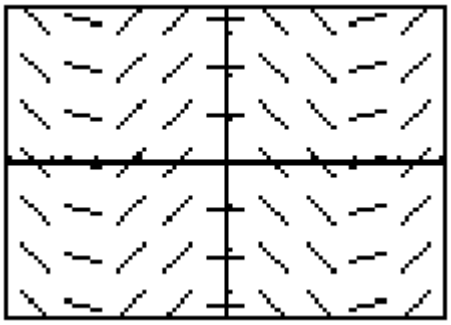


For 7 – 12, match each slope field with the **equation** that the slope field could represent.

7.



8.



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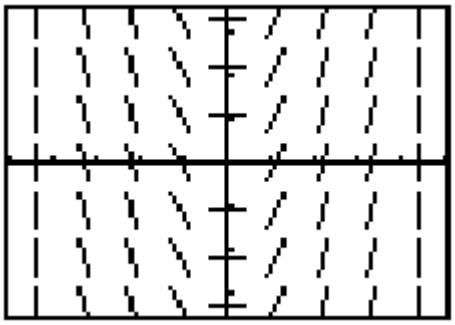

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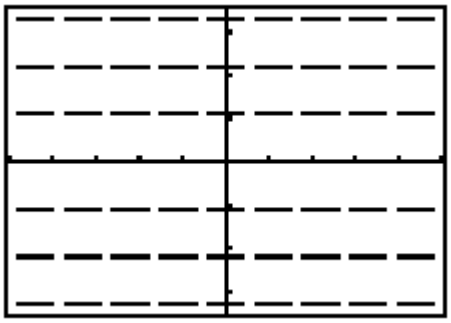

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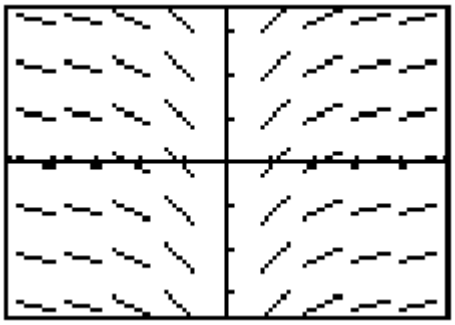
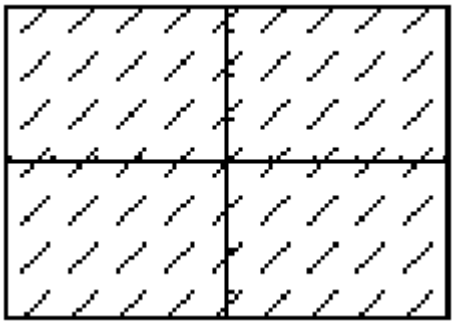


12.



(A) $y = 1$

(B) $y = x$

(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$

(E) $y = \frac{1}{x^2}$

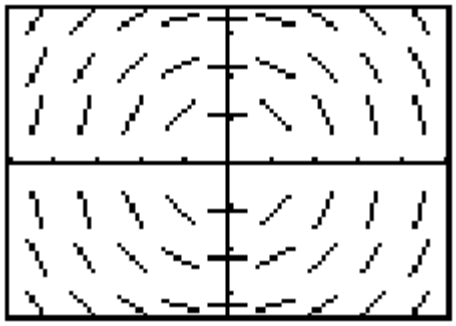
(F) $y = \sin x$

(G) $y = \cos x$

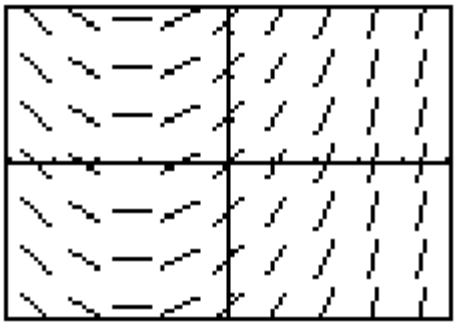
(H) $y = \ln|x|$

For 13 – 16, match the slope fields with their differential equations.

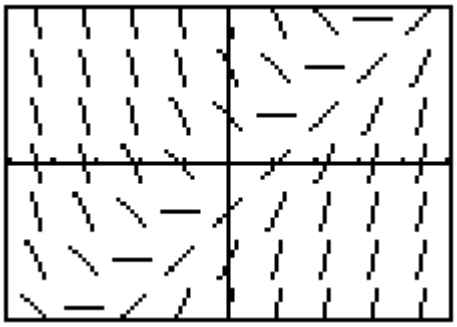
13.



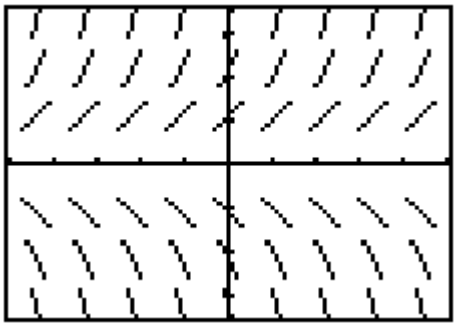
14.



15.



16.



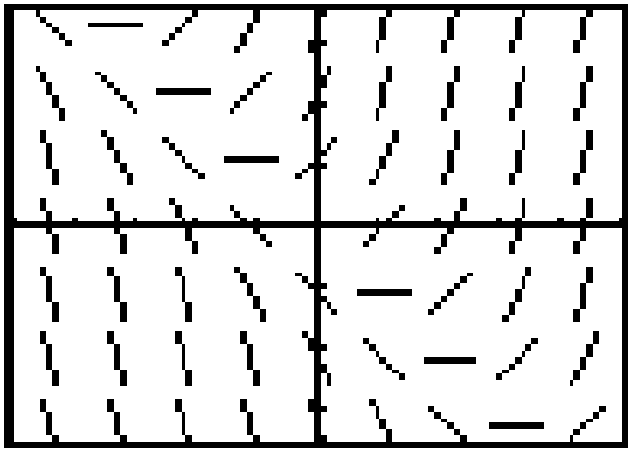
(A) $\frac{dy}{dx} = \frac{1}{2}x + 1$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = y$

(D) $\frac{dy}{dx} = -\frac{x}{y}$

17. The calculator-drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.



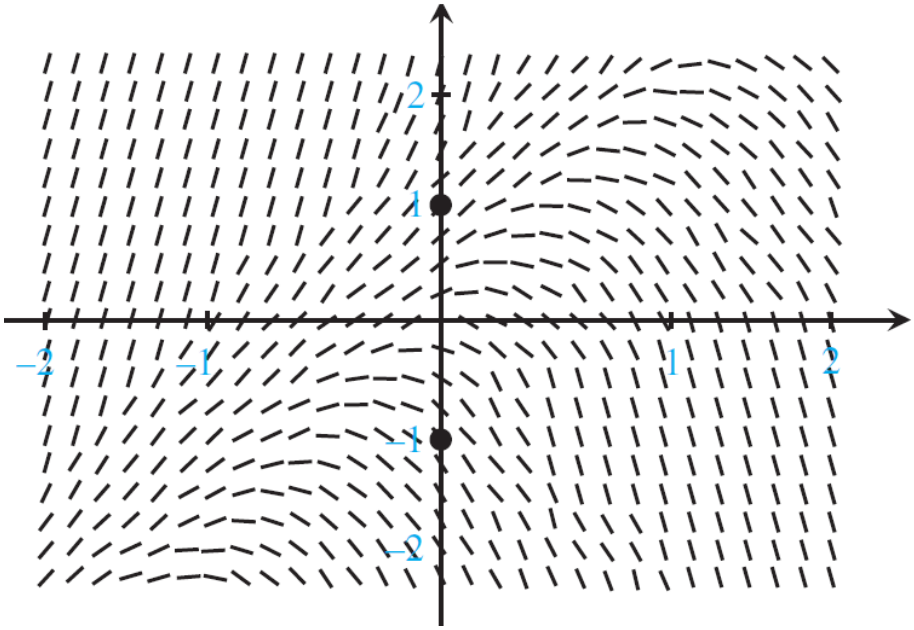
(a) Sketch the solution curve through the point $(0,1)$.

(b) Sketch the solution curve through the point $(-3,0)$.

(c) Approximate $y(-3.1)$ using the equation of the tangent line to $y = f(x)$ at the point $(-3,0)$.

18. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the differential equation is shown below. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that goes through the point $(0,-1)$.

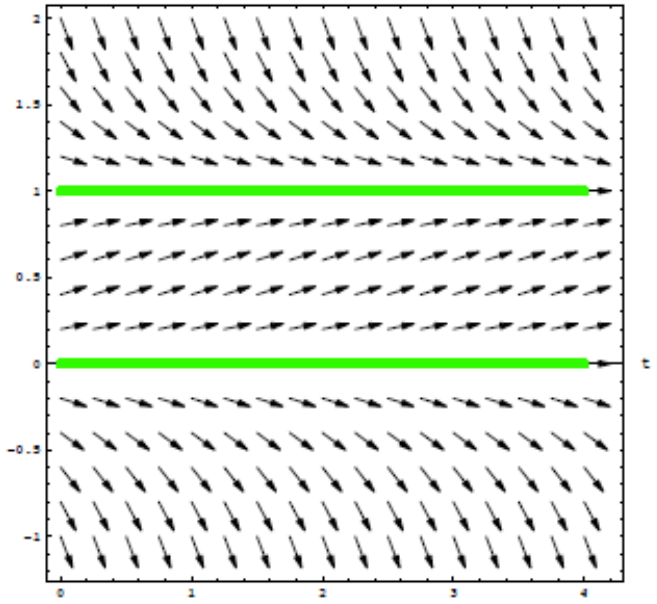


(b) There is a value of b for which $y = 2x + b$ is a solution to the differential equation. Find this value of b. Justify your answer.

(c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. It appears from the slope field that g has a local maximum at the point $(0,0)$. Using the differential equation, prove analytically that this is so.

Multiple Choice:

19. Given the following slope field (with **equilibrium solutions**, meaning the slopes are zero and a the existence of a horizontal asymptote on the solution graph, at $y = 0$ and $y = 1$), find the matching differential equation.



(A) $\frac{dy}{dx} = y(y-1)$ (B) $\frac{dy}{dx} = y(1-y)$

(C) $\frac{dy}{dx} = \frac{1}{y(1-y)}$ (D) $\frac{dy}{dx} = 1 - e^{y(1-y)}$

(E) $\frac{dy}{dx} = \frac{y}{y-1}$

20. A slope field for the differential equation $\frac{dy}{dx} = 42 - y$ will show

(A) a line with slope -1 and y-intercept of 42 (B) a vertical asymptote at $x = 42$

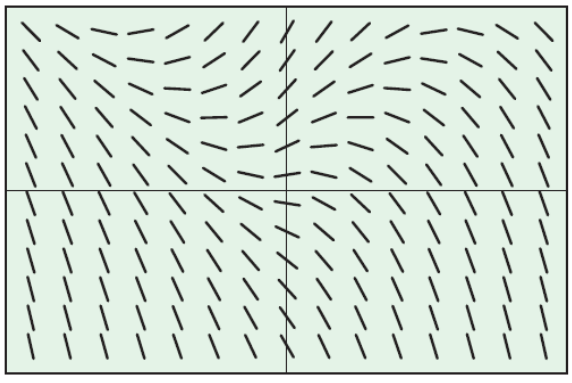
(C) a horizontal asymptote at $y = 42$ (D) a family of parabolas opening downward

(E) a family of parabolas opening to the left

21. For which of the following differential equations will a slope field show nothing but negative slopes in the fourth quadrant?

(A) $\frac{dy}{dx} = -\frac{x}{y}$ (B) $\frac{dy}{dx} = xy + 5$ (C) $\frac{dy}{dx} = xy^2 - 2$ (D) $\frac{dy}{dx} = \frac{x^3}{y^2}$ (E) $\frac{dy}{dx} = \frac{y}{x^2} - 3$

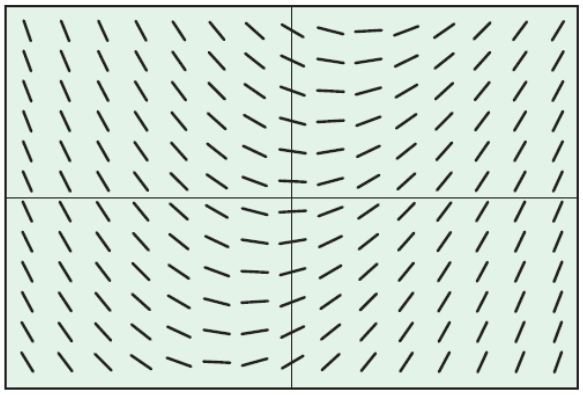
22. Which of the following differential equations would produce the slope field shown below?



[–3, 3] by [–1.98, 1.98]

(A) $\frac{dy}{dx} = y - |x|$ (B) $\frac{dy}{dx} = |y| - x$ (C) $\frac{dy}{dx} = |y - x|$ (D) $\frac{dy}{dx} = |y + x|$ (E) $\frac{dy}{dx} = |y| - |x|$

23. Which of the following differential equations would produce the slope field shown below?

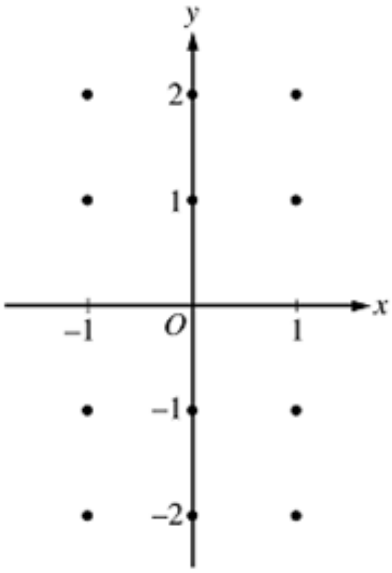


(A) $\frac{dy}{dx} = y - 3x$ (B) $\frac{dy}{dx} = y - \frac{x}{3}$ (C) $\frac{dy}{dx} = y + \frac{x}{3}$ (D) $\frac{dy}{dx} = x + \frac{y}{3}$ (E) $\frac{dy}{dx} = x - \frac{y}{3}$

24. AP 2010B-5 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided at right, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.



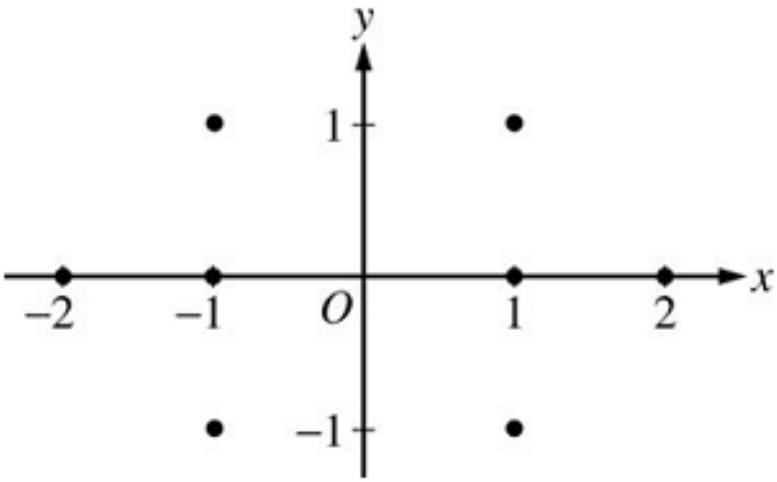
(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane for which $y \neq 0$. Describe all points in the *xy*-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

25. AP 2006-5 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

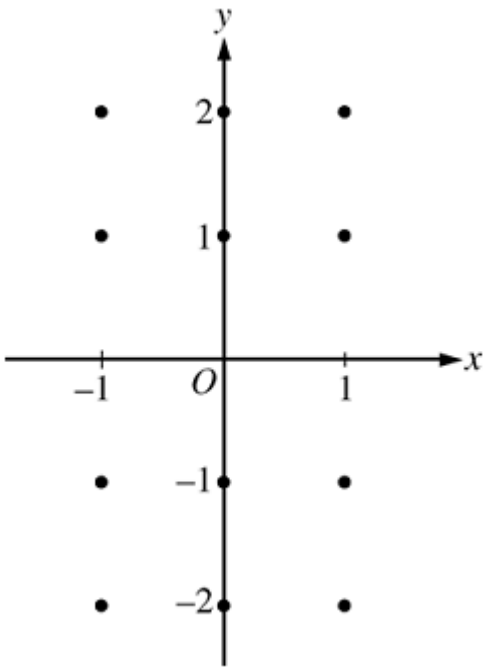


(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

26. AP 2005-6 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1,-1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.