Name KEY Date TODAY Period $\sqrt{-1}$

## Mixed Integration Worksheet

**Part I**:
For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int (x^3+1)\,dx$

$= \frac{1}{4}x^4+x+C$

(nothing)

$\int x^2(x^3+1)^4\,dx$

$= \frac{1}{3}\left(\frac{1}{5}\right)(x^3+1)^5+C$

corr pwr rule

$= \frac{1}{15}(x^3+1)^5+C$

(substitution)

$\int \sqrt{x^3+1}\,dx$

(Can't do)

$\int (x^3+1)^2\,dx$

$= \int (x^6+2x^3+1)\,dx$

$= \frac{1}{7}x^7+\frac{1}{2}x^4+x+C$

(algebra)

B. $\int \sqrt{x}(1-x^2)\,dx$

$\int (x^{1/2}-x^{5/2})\,dx$

$\frac{2}{3}x^{3/2}-\frac{2}{7}x^{7/2}+C$

(algebra)

$\int \sqrt{1-x^2}\,dx$

(can't do)

$\int \frac{1}{\sqrt{1-x^2}}\,dx$

$= \arcsin x + C$

(nothing)

$\int \frac{x\,dx}{\sqrt{1-x^2}}$

$= \int x(1-x^2)^{-1/2}\,dx$

$\left(-\frac{1}{2}\right)(2)(1-x^2)^{1/2}+C$

$-\sqrt{1-x^2}+C$

(substitution)

C. $\int \cos^2 x\sin^3 x\,dx$

$\int \cos^2 x(\sin^2 x)\sin x\,dx$

$\int \cos^2 x\sin x(1-\cos^2 x)\,dx$

$\int (\cos^2 x\sin x-\cos^4 x\sin x)\,dx$

$= -\frac{1}{3}\cos^3 x+\frac{1}{5}\cos^5 x+C$

(trig sub/sub)

$\int \sqrt{1-\cos^2 x}\,dx$

$\int \sqrt{\sin^2 x}\,dx$

$\int \sin x\,dx$

$-\cos x+C$

(trig sub/alg)

$\int \frac{dx}{\cos^2 x}$

$\int \sec^2 x\,dx$

$\tan x+C$

(trig sub/nothing)

$\int \frac{dx}{\cos x\sqrt{\sin x}}$

(can't do)

D. $\int \tan x\sec x\,dx$

$= \sec x+C$

(nothing)

$\int \tan x\cos x\,dx$

$\int \frac{\sin x}{\cancel{\cos x}}\cancel{\cos x}\,dx$

$\int \sin x\,dx$

$-\cos x+C$

(trig id)

$\int \frac{\sec^2 x}{\sqrt{\tan x}}\,dx$

$\int \sec^2 x\cdot(\tan x)^{-1/2}\,dx$

$2(\tan x)^{1/2}+C$

$2\sqrt{\tan x}+C$

(substitution)

$\int \frac{dx}{\tan x+1}$

(can't do)

E. $\int e^{-x^2}\,dx$

(Can't do)

$\int \frac{e^x}{3+e^x}\,dx$

$\int e^x(3+e^x)^{-1}\,dx$

$\ln|3+e^x|+C$

(substitution)

$\int (e^x+3)\,dx$

$= e^x+3x+C$

(nothing)

$\int \frac{\ln(e^{2x})}{x^2}\,dx$

$= \int \frac{2x}{x^2}\,dx$

$= \int 2\left(\frac{1}{x}\right)dx$

$= 2\ln|x|+C$

(algebra)

**Part II**: Evaluate the integrals

1. $\int (5x+4)^5\,dx$

$= \left(\frac{1}{5}\right)\left(\frac{1}{6}\right)(5x+4)^6 + C$

$= \frac{1}{30}(5x+4)^6 + C$

2. $\int 3t^2\left(t^3+4\right)^5\,dt$

$= \frac{1}{6}(t^3+4)^6 + C$

3. $\int \sqrt{4x-5}\,dx$

$= \left(\frac{1}{4}\right)\left(\frac{2}{3}\right)(4x-5)^{3/2} + C$

$= \frac{1}{6}(4x-5)^{3/2} + C$

4. $\int t^2\left(t^3+4\right)^{-1/2}\,dt$

$= \left(\frac{1}{3}\right)(2)(t^3+4)^{1/2} + C$

$= \frac{2}{3}\sqrt{t^3+4} + C$

5. $\int \cos(2x+1)\,dx$

$= \frac{1}{2}\sin(2x+1) + C$

6. $\int \sin^{10} x \cos x\,dx$

$\frac{1}{11}\sin^{11} x + C$

7. $\int \frac{\sin x}{\cos^5 x}\,dx$

$= \int \sin x\,(\cos x)^{-5}\,dx$

$= (-1)\left(-\frac{1}{4}\right)(\cos x)^{-4} + C$

$= \frac{1}{4\cos^4 x} + C$

8. $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}}\,dx$

$= 2\left(\frac{1}{3}\right)(\sqrt{x}-1)^3 + C$

$= \frac{2}{3}(\sqrt{x}-1)^3 + C$

9. $\int \sqrt{x^3+x^2}\left(3x^2+2x\right)dx$

$= \frac{2}{3}(x^3+x^2)^{3/2} + C$

10. $\int \frac{x+1}{\left(x^2+2x+2\right)^3}\,dx$

$= \frac{1}{2}\left(-\frac{1}{2}\right)(x^2+2x+2)^{-2} + C$

$= -\frac{1}{4(x^2+2x+2)^2} + C$

11. $\int \cos(2x)\sqrt{\sin(2x)}\,dx$

$= \frac{1}{2}\left(\frac{2}{3}\right)(\sin 2x)^{3/2} + C$

$= \frac{1}{3}(\sin 2x)^{3/2} + C$

12. $\int (x+1)\sin\left(x^2+2x+3\right)dx$

$= -\frac{1}{2}\cos(x^2+2x+3) + C$

13. $\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt$

$= (-1)\left(\frac{1}{4}\right)\left(1+\frac{1}{t}\right)^4 + C$

$= -\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C$

14. $\int x^2\sqrt{x^3+1}\,dx$

$= \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(x^3+1)^{3/2} + C$

$= \frac{2}{9}(x^3+1)^{3/2} + C$

15. $\int \frac{2}{\sqrt{3x-7}}\,dx$

$= \int 2(3x-7)^{-1/2}\,dx$

$= 2\left(\frac{1}{3}\right)(2)(3x-7)^{1/2} + C$

$= \frac{4}{3}\sqrt{3x-7} + C$

16. $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}\,dx$

$= \int \frac{1}{\sqrt{x}}(\sqrt{x}+1)^{-2}\,dx$

$= 2(-1)(\sqrt{x}+1)^{-1} + C$

$= \frac{-2}{\sqrt{x}+1} + C$

17. $\int \frac{x}{\sqrt{x+1}}\,dx$ * u-sub

$u = x+1 \quad x = u-1$

$du = dx,$

$= \int (u-1)u^{-1/2}\,du$

$= \int \left(u^{1/2} - u^{-1/2}\right)du$

$= \frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$

18. $\int x\sqrt{2x+1}\,dx$ * u-sub

$u = 2x+1, \quad x = \frac{1}{2}(u-1)$

$du = 2dx,$

$dx = \frac{1}{2}du$

$= \left(\frac{1}{2}\right)\frac{1}{2}\int (u-1)u^{1/2}\,du$

$= \frac{1}{4}\int \left(u^{3/2} - u^{1/2}\right)du$

$= \frac{1}{4}\left[\frac{2}{5}(2x+1)^{5/2} - \frac{2}{3}(2x+1)^{3/2}\right] + C$

19. $\int \sqrt{x}\sqrt{x\sqrt{x}+1}dx$

$= \int x^{1/2}(x^{3/2}+1)^{1/2}dx$

$= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)(x^{3/2}+1)^{3/2}+C$

$= \frac{4}{9}(x^{3/2}+1)^{3/2}+C$

20. $\int x\tan\left(x^2\right)\sec\left(x^2\right)dx$

$= \frac{1}{2}\sec(x^2)+C$

21. $\int\left(x^2+1\right)\sqrt{x-2}dx$ *u-sub $u=x-2$, $x=u+2$, $du=dx$

$= \int((u+2)^2+1)u^{1/2}du$

$= \int(u^2+4u+5)u^{1/2}du = \int(u^{5/2}+4u^{3/2}+5u^{1/2})du$

$= \frac{2}{7}(x-2)^{7/2}+\frac{8}{5}(x-2)^{5/2}+\frac{10}{3}(x-2)^{3/2}+C$

22. $\int\frac{x^2+2x}{x^2+2x+1}dx$ *Long divide

$$\begin{array}{r} 1 \\ x^2+2x+1\,\overline{)\,x^2+2x} \\ \underline{x^2+2x+1} \\ -1 \end{array}$$

$= \int\left(1-\frac{1}{(x+1)^2}\right)dx$

$= \int(1-(x+1)^{-2})dx$

$= x+(x+1)^{-1}+C$

$= x+\frac{1}{x+1}+C$

23. $\int\frac{1}{x^2+6x+9}dx$

$= \int\frac{1}{(x+3)^2}dy$

$= \int(x+3)^{-2}dx$

$= \frac{-1}{x+3}+C$

24. $\int\frac{\sec^2 x}{(1+\tan x)^3}dx$

$= \int\sec^2x(1+\tan x)^{-3}dx$

$= -\frac{1}{2}(1+\tan x)^{-2}+C$

$= \frac{-1}{2(1+\tan x)^2}+C$

25. $\int\frac{\sin x}{(2+3\cos x)^2}dx$

$= \int\sin x(2+3\cos x)^{-2}dx$

$= -3(-1)(2+3\cos x)^{-1}+C$

$= \frac{3}{2+3\cos x}+C$

26. $\int x\tan^2\left(x^2\right)\sec^2\left(x^2\right)dx$

$= \int x\cdot\sec^2(x^2)\cdot(\tan(x^2))^2dx$

$= \frac{1}{2}\left(\frac{1}{3}\right)(\tan(x^2))^3+C$

$= \frac{1}{6}\tan^3(x^2)+C$

27. $\int(\tan 2x+\cot 2x)^2dx$

$= \int(\tan^2 2x+2\tan 2x\cdot\cot 2x+\cot^2 2x)dx$

$= \int(\sec^2 2x-1+2+\csc^2 2x-1)dx$

$= \int(\sec^2 2x+\csc^2 2x)dx$

$= \frac{1}{2}\tan 2x-\frac{1}{2}\cot 2x+C$

28. $\int\frac{xe^{x^2}}{e^{x^2}+1}dx$

$= \frac{1}{2}\ln|e^{x^2}+1|+C$

or $= \frac{1}{2}\ln(e^{x^2}+1)+C$

29. $\int\frac{1}{\sqrt{-x^2+5x-6}}dx$ *complete the square

$= \int\frac{1}{\sqrt{-(x^2-5x+\frac{25}{4})+\frac{25}{4}-6}}$

$= \int\frac{1}{\sqrt{\frac{1}{4}-(x-\frac{5}{2})^2}}dx$, $a=\frac{1}{2}$, $u=x-\frac{5}{2}$

$= \arcsin\left(\frac{x-\frac{5}{2}}{1/2}\right)+C = \arcsin(2x-5)+C$

30. $\int\frac{x}{1+x^2}dx$

$= \frac{1}{2}\ln|1+x^2|+C$

or $= \frac{1}{2}\ln(1+x^2)+C$

31. $\int\frac{4}{5x\sqrt{x^2-3}}dx$

$= \frac{4}{5}\int\frac{4}{x\sqrt{x^2-(\sqrt{3})^2}}dx$

$= \frac{4}{5}\left(\frac{1}{\sqrt{3}}\right)\text{arcsec}\frac{|x|}{\sqrt{3}}+C$

$= \frac{4}{5\sqrt{3}}\text{arcsec}\frac{|x|}{\sqrt{3}}+C$

32. $\int\frac{x^2}{1+x^2}dx$ *Long divide

$$\begin{array}{r} 1 \\ x^2+1\,\overline{)\,x^2} \\ \underline{x^2+1} \\ -1 \end{array}$$

$= \int\left(1-\frac{1}{x^2+1}\right)dx$

$= x-\arctan x+C$

33. $\int xe^{x^2}dx$

$= \frac{1}{2}e^{x^2}+C$

34. $\int\frac{x}{\sqrt{x-1}}dx$ *u-sub $u=x-1$, $x=u+1$, $du=dx$

$= \int(u+1)u^{-1/2}du$

$= \int(u^{1/2}+u^{-1/2})du$

$= \frac{2}{3}(x-1)^{3/2}+2(x-1)^{1/2}+C$

35. $\int\left(6x+\frac{7}{\sqrt{9-x^2}}\right)dx$

$= \int\left(6x+7\left(\frac{1}{\sqrt{3^2-x^2}}\right)\right)dx$

$= 3x^2+7\arcsin\left(\frac{x}{3}\right)+C$

36. $\int x^2\sqrt{x+1}dx$ *u-sub $u=x+1$, $x=u-1$, $du=dx$

$\int(u-1)^2u^{1/2}du$

$\int(u^2-2u+1)u^{1/2}du$

$\int(u^{5/2}-2u^{3/2}+u^{1/2})du$

$= \frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}+C$

37. $\int \left(1+e^{-x}\right)^2 dx$

$$= \int \left(1+2e^{-x}+e^{-2x}\right)dx$$
$$= x - 2e^{-x} - \frac{1}{2}e^{-2x} + C$$

38. $\int \frac{6\cos x - 2\sin x}{6\sin x + 2\cos x}dx$

$$= \ln\left|6\sin x + 2\cos x\right| + C$$

39. $\int \frac{4}{x}\sqrt[3]{(1+2\ln x)^2}\,dx$

$$= 4\int \left(\frac{1}{x}\right)(1+2\ln x)^{2/3}dx$$
$$= 4\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)(1+2\ln x)^{5/3} + C$$
$$= \frac{6}{5}(1+2\ln x)^{5/3} + C$$

40. $\int \frac{2e^{\tan x}+5}{\cos^2 x}dx$

$$= \int \sec^2 x\left(2e^{\tan x}+5\right)dx$$
$$= \int \left(2\sec^2 x\, e^{\tan x} + 5\sec^2 x\right)dx$$
$$= 2e^{\tan x} + 5\tan x + C$$

41. $\int \frac{\left(1-x^2\right)^{-1/2}}{3+2\arcsin x}dx$

$$= \int \left(\frac{1}{\sqrt{1-x^2}}\right)\left(\frac{1}{3+2\arcsin x}\right)dx$$
$$= \frac{1}{2}\ln\left|3+2\arcsin x\right| + C$$

42. $\int \frac{t^3}{\sqrt{1-t^8}}dt$

$$= \int t^3\left(\frac{1}{\sqrt{1-(t^4)^2}}\right)dt$$
$$= \frac{1}{4}\arcsin t^4 + C$$

43. $\int \frac{5-x}{\sqrt{4-5x^2}}dx$

$$= \int \left(5\left(\frac{1}{\sqrt{2^2-(\sqrt{5}x)^2}}\right) - x\left(4-5x^2\right)^{-1/2}\right)dx$$
$$= \frac{5}{\sqrt{5}}\arcsin\frac{\sqrt{5}x}{2} + \frac{1}{10}(2)\left(4-5x^2\right)^{1/2} + C$$
$$= \frac{5}{\sqrt{5}}\arcsin\left(\frac{\sqrt{5}}{2}x\right) + \frac{1}{5}\sqrt{4-5x} + C$$

**Part III**: Solve the differential equations. If no initial value is indicated, find the general solution.

44. Find the value of $y\left(\frac{5\pi}{3}\right)$ when $\frac{dy}{d\theta} = \cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)$ and $y(0)=0$.

$$\int dy = \int \left(\cos\left(\frac{\theta}{5}\right)\right)^2 \cdot \sin\left(\frac{\theta}{5}\right)d\theta$$
$$y = -\frac{1}{3}(5)\cos^3\left(\frac{\theta}{5}\right) + C$$
$$\text{at } (0,0):\ 0 = -\frac{5}{3} + C,\ C = \frac{5}{3}$$
$$\text{So } y = -\frac{5}{3}\cos^3\left(\frac{\theta}{5}\right) + \frac{5}{3}$$
$$y\left(\frac{5\pi}{3}\right) = -\frac{5}{3}\left(\cos\left(\frac{\pi}{3}\right)\right)^3 + \frac{5}{3}$$
$$y\left(\frac{5\pi}{3}\right) = -\frac{5}{3}\left(\frac{1}{8}\right) + \frac{5}{3}$$
$$= -\frac{5}{24} + \frac{40}{24}$$
$$= \boxed{\frac{35}{24}}$$

45. Find the value of $y(\pi)$ when $\frac{dy}{dx} = 8e^{-2x} - 2\sin x$ and $y(0)=4$

$$\int dy = \int \left(8e^{-2x} - 2\sin x\right)dx$$
$$y = -4e^{-2x} + 2\cos x + C$$
$$\text{at } (0,4):\ 4 = -4 + 2 + C,\ C = 6$$
$$\text{So } y = -4e^{-2x} + 2\cos x + 6$$
$$y(\pi) = -4e^{-2\pi} + 2\cos\pi + 6$$
$$y(\pi) = \boxed{-4e^{-2\pi} + 4}$$

46. Find the value of $f(-1)$ when $f'(x) = 6xe^{-2x^2}$ and $f(0)=1$.

$$f(x) = -\frac{6}{4}e^{-2x^2} + C$$
$$f(x) = -\frac{3}{2}e^{-2x^2} + C$$
$$\text{at } (0,1):\ 1 = -\frac{3}{2} + C,\ C = \frac{5}{2}$$
$$\text{So } f(x) = -\frac{3}{2}e^{-2x^2} + \frac{5}{2}$$
$$f(-1) = \boxed{-\frac{3}{2}e^{-2} + \frac{5}{2}}$$

47. $\frac{dy}{dt} = (t+1)e^{\frac{5}{2}t^2+5t}$

$$\int dy = \int (t+1)e^{\frac{5}{2}t^2+5t}dt$$
$$y = \frac{1}{5}e^{\frac{5}{2}t^2+5t} + C$$

48. $f'(x) = \frac{1+e^{3x}}{e^{3x}+3x}$

$$f(x) = \frac{1}{3}\ln\left|e^{3x}+3x\right| + C$$

49. $y' = \frac{\sin(\ln 5x)}{x}$

$$y = -\cos(\ln 5x) + C$$

50. $\frac{dy}{dx}=\frac{1}{1+9x^2}$ where $y\left(\frac{1}{3}\right)=2$

$\int dy = \int \frac{1}{1+(3x)^2}\,dx$

$y = \frac{1}{3}\arctan(3x) + C$

at $\left(\frac{1}{3},2\right)$: $2 = \frac{1}{3}\arctan 1 + C$

$2 = \frac{\pi}{12} + C,\ C = 2 - \frac{\pi}{12}$

$y = \frac{1}{3}\arctan(3x) + 2 - \frac{\pi}{12}$

51. $\frac{dy}{dx}=\left(1+y^2\right)\tan x$ if $y(0)=\sqrt{3}$

$\frac{1}{1+y^2}dy = \int \tan x\,dx$

$\arctan y = -\ln|\cos x| + C$

at $(0,\sqrt{3})$: $\arctan\sqrt{3} = -\ln|\cos 0| + C$

$C = \frac{\pi}{3}$ so $y = \tan\left(-\ln|\cos x| + \frac{\pi}{3}\right)$

52. $\frac{dy}{dx}=1$

$\int dy = \int dx$

$y = x + C$

53. $\frac{dy}{dx} - yx = 0$

$\frac{dy}{dx} = yx$

$\frac{1}{y}dy = x\,dx$

$\ln|y| = \frac{1}{2}x^2 + C$

$y = \pm e^{1/2x^2+C} = Ce^{\frac{1}{2}x^2}$

54. $e^y\frac{dy}{dx}=1$

$\int e^y dy = \int dx$

$e^y = x + C$

$y = \ln(x+C)$

55. $y^2x^2\frac{dy}{dx}=x$

$y^2 dy = \frac{1}{x}dx$

$\frac{1}{3}y^3 = \ln|x| + C$

$y^3 = 3\ln|x| + C$

$y = \sqrt[3]{3\ln|x| + C}$

**Part IV**: Challenging ones

56. $\int \frac{7}{\sqrt{x}\sqrt{2-x}}dx$ * u-sub $u=\sqrt{x},\ x=u^2$, $dx = 2u\,du$

$= \int \frac{7}{\cancel{u}\sqrt{2-u^2}}\cdot 2\cancel{u}\,du = 14\int\frac{1}{\sqrt{(\sqrt{2})^2-u^2}}du$

$= 14\arcsin\frac{u}{\sqrt{2}} + C$

$= 14\arcsin\sqrt{\frac{x}{2}} + C$

57. $\int_0^1 \frac{x^2+4x+1}{3x^2+3}dx$ * Long div

$$\begin{array}{r|l} & 1 \\ x^2+3 & x^2+4x+1 \\ & -x^2+3 \\ \hline & 4x-2 \end{array}$$

$= \frac{1}{3}\int\left(1 + \frac{4x-2}{x^2+3}\right)dx$

$= \frac{1}{3}\int\left(1 + 4\left(\frac{x}{x^2+3}\right) - 2\left(\frac{1}{x^2+3}\right)\right)dx$

$= \frac{1}{3}\left[x + 2\ln|x^2+3| - 2\left(\frac{1}{\sqrt{3}}\right)\arctan\frac{x}{\sqrt{3}}\right]\Big|_0^1$

$= \frac{1}{3}\left[\left(1 + 2\ln 4 - \frac{2}{\sqrt{3}}\left(\frac{\pi}{6}\right)\right) - (2\ln 3)\right]$

58. $\int_0^{\pi/2}\left(2\sin\theta - \sin^3\theta\right)d\theta$

$\int(2\sin\theta - \sin\theta(1-\cos^2\theta))d\theta$

$= \int(2\sin\theta - \sin\theta + \sin\theta(\cos\theta)^2)d\theta$

$= \int(\sin\theta + \sin\theta(\cos\theta)^2)d\theta$

$= -\cos\theta - \frac{1}{3}\cos^3\theta\Big|_0^{\pi/2}$

$(0) - \left(-1 - \frac{1}{3}\right) = \boxed{\frac{4}{3}}$

59. $\int_0^{\pi/4}\frac{3\cos x - 4\sin x}{\cos^3 x}dx$

$= \int\left(3\frac{\cos x}{\cos^3 x} - \frac{4\sin x}{\cos^3 x}\right)dx$

$= \int(3\sec^2 x - 4\tan x\cdot\sec^2 x)dx$

$= 3\tan x - 2\tan^2 x\Big|_0^{\pi/4}$

$= (3-2) - (0)$

$= \boxed{1}$

60. $\int 3t^3\left(t^2+4\right)^5 dt$ * u-sub $u = t^2+4,\ t^2 = u-4$; $du = 2t\,dt,\ t^3dt = t^2\cdot t\,dt$; $t\,dt = \frac{1}{2}du$

$= \frac{3}{2}\int(u-4)u^5 du$

$= \frac{3}{2}\int(u^6 - 4u^5)du$

$= \frac{3}{2}\left[\frac{1}{7}(t^2+4)^7 - \frac{2}{3}(t^2+4)^6\right] + C$

61. $\int x^3\sqrt{x^2-1}\,dx$ * usub $x^2-1=u,\ x^2=u+1$; $x\,dx = \frac{1}{2}du$

$= \int x^2\cdot(x^2-1)^{1/2}\cdot x\,dx$

$= \frac{1}{2}\int(u+1)u^{1/2}du$

$= \frac{1}{2}\int\left(u^{3/2} + u^{1/2}\right)du$

$= \frac{1}{2}\left[\frac{2}{5}(x^2-1)^{5/2} + \frac{2}{3}(x^2-1)^{3/2}\right] + C$

62. $\int\left(1+e^{-x}\right)^{-1}dx$

$= \int\frac{1}{1+e^{-x}}\left(\frac{e^x}{e^x}\right)dx$

$= \int\frac{e^x}{1+e^x}dx$

$= \ln|1+e^x| + C$

63. $\int_1^e\frac{1}{x}\left[f'(\ln x)+2\right]dx$ when $f(0)=1$ and $f(1)=4$

$= \int\left(\frac{1}{x}f'(\ln x) + 2\left(\frac{1}{x}\right)\right)dx$

$= f(\ln x) + 2\ln|x|\Big|_1^e$

$= (f(\ln e) + 2\ln e) - (f(\ln 1) + 2\ln 1)$

$= f(1) + 2 - f(0) + 2$

$= 4 + 2 - 1 + 2$

$= \boxed{7}$