Name_____ Date_____ Period_____

Worksheet 5.2—Mega Integration Worksheet (AB Methods)

Part I:
For each integral decide which of the following is needed: 1) substitution, 2) algebra or a trig identity, 3) nothing needed, or 4) can't be done by the techniques in Calculus I. Then evaluate each integral (except for the 4th type of course).

A. $\int \left(x^3+1\right)dx$ $\int x^2\left(x^3+1\right)^4 dx$ $\int \sqrt{x^3+1}dx$ $\int \left(x^3+1\right)^2 dx$

B. $\int \sqrt{x}\left(1-x^2\right)dx$ $\int \sqrt{1-x^2}dx$ $\int \frac{1}{\sqrt{1-x^2}}dx$ $\int \frac{xdx}{\sqrt{1-x^2}}$

C. $\int \cos^2 x \sin^3 x dx$ $\int \sqrt{1-\cos^2 x}dx$ $\int \frac{dx}{\cos^2 x}$ $\int \frac{dx}{\cos x\sqrt{\sin x}}$

D. $\int \tan x \sec x dx$ $\int \tan x \cos x dx$ $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$ $\int \frac{dx}{\tan x + 1}$

E. $\int e^{-x^2} dx$ $\int \frac{e^x}{3+e^x} dx$ $\int \left(e^x + 3\right) dx$ $\int \frac{\ln\left(e^{2x}\right)}{x^2} dx$

Part II: Evaluate the integrals

1. $\int \left(5x+4\right)^5 dx$
2. $\int 3t^2 \left(t^3+4\right)^5 dt$
3. $\int \sqrt{4x-5} dx$
4. $\int t^2 \left(t^3+4\right)^{-1/2} dt$
5. $\int \cos\left(2x+1\right) dx$
6. $\int \sin^{10} x \cos x dx$

7. $\int \frac{\sin x}{\cos^5 x}\,dx$

8. $\int \frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}}\,dx$

9. $\int \sqrt{x^3+x^2}\left(3x^2+2x\right)dx$

10. $\int \frac{x+1}{\left(x^2+2x+2\right)^3}\,dx$

11. $\int \cos 2x\sqrt{\sin 2x}\,dx$

12. $\int (x+1)\sin\left(x^2+2x+3\right)dx$

13. $\int\left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt$

14. $\int x^2\sqrt{x^3+1}\,dx$

15. $\int \frac{2}{\sqrt{3x-7}}\,dx$

16. $\int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^2}\,dx$

17. $\int \frac{x}{\sqrt{x+1}}\,dx$

18. $\int x\sqrt{2x+1}\,dx$

19. $\int \sqrt{x}\sqrt{x\sqrt{x}+1}dx$

20. $\int x\tan\left(x^2\right)\sec\left(x^2\right)dx$

21. $\int\left(x^2+1\right)\sqrt{x-2}dx$

22. $\int \frac{x^2+2x}{x^2+2x+1}dx$

23. $\int \frac{1}{x^2+6x+9}dx$

24. $\int \frac{\sec^2 x}{\left(1+\tan x\right)^3}dx$

25. $\int \frac{\sin x}{\left(2+3\cos x\right)^2}dx$

26. $\int x\tan^2\left(x^2\right)\sec^2\left(x^2\right)dx$

27. $\int\left(\tan 2x+\cot 2x\right)^2 dx$

28. $\int \frac{xe^{x^2}}{e^{x^2}+1}dx$

29. $\int \frac{1}{\sqrt{-x^2+5x-6}}dx$

30. $\int \frac{x}{1+x^2}dx$

31. $\int \frac{4}{5x\sqrt{x^2-3}}dx$

32. $\int \frac{x^2}{1+x^2}dx$

33. $\int xe^{x^2}dx$

34. $\int \frac{x}{\sqrt{x-1}}dx$

35. $\int\left(6x+\frac{7}{\sqrt{9-x^2}}\right)dx$

36. $\int x^2\sqrt{x+1}dx$

37. $\int\left(1+e^{-x}\right)^2dx$

38. $\int \frac{6\cos x-2\sin x}{6\sin x+2\cos x}dx$

39. $\int \frac{4}{x}\sqrt[3]{(1+2\ln x)^2}dx$

40. $\int \frac{2e^{\tan x}+5}{\cos^2 x}dx$

41. $\int \frac{\left(1-x^2\right)^{-1/2}}{3+2\arcsin x}dx$

42. $\int \frac{t^3}{\sqrt{1-t^8}}dt$

43. $\int \frac{5-x}{\sqrt{4-5x^2}}dx$

Part III: Solve the differential equations. If no initial value is indicated, find the general solution.

44. Find the value of $y\left(\frac{5\pi}{3}\right)$ when $\frac{dy}{d\theta}=\cos^2\left(\frac{\theta}{5}\right)\sin\left(\frac{\theta}{5}\right)$ and $y(0)=0$.

45. Find the value of $y(\pi)$ when $\frac{dy}{dx}=8e^{-2x}-2\sin x$ and $y(0)=4$

46. Find the value of $f(-1)$ when $f'(x)=6xe^{-2x^2}$ and $f(0)=1$.

47. $\frac{dy}{dt}=(t+1)e^{\frac{5}{2}t^2+5t}$

48. $f'(x)=\frac{1+e^{3x}}{e^{3x}+3x}$

49. $y'=\frac{\sin(\ln 5x)}{x}$

50. $\frac{dy}{dx} = \frac{1}{1+9x^2}$ where $y\left(\frac{1}{3}\right) = 2$

51. $\frac{dy}{dx} = \left(1+y^2\right)\tan x$ if $y(0) = \sqrt{3}$

52. $\frac{dy}{dx} = 1$

53. $\frac{dy}{dx} - yx = 0$

54. $e^y \frac{dy}{dx} = 1$

55. $y^2 x^2 \frac{dy}{dx} = x$

Part IV: Challenging ones

56. $\int \frac{7}{\sqrt{x}\sqrt{2-x}} dx$

57. $\int_0^1 \frac{x^2+4x+1}{3x^2+3} dx$

58. $\int_0^{\pi/2} \left(2\sin\theta - \sin^3\theta\right) d\theta$

59. $\int_0^{\pi/4} \frac{3\cos x - 4\sin x}{\cos^3 x}\,dx$

60. $\int 3t^3\left(t^2+4\right)^5\,dt$

61. $\int x^3\sqrt{x^2-1}dx$

62. $\int\left(1+e^{-x}\right)^{-1}\,dx$

63. $\int_1^e \frac{1}{x}\left[\,f'\left(\ln x\right)+2\right]dx$ when $f\left(0\right)=1$ and $f\left(1\right)=4$