Name____ Date_________________________ Period______

Worksheet 5.3—Euler's Method
Show all work. Unless stated, you MAY use a calculator, but show all steps.

1. Answer the following questions.

(a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch your solution.

(b) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

(c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

2. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2)=5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		

3. Given the differential equation $\dfrac{dy}{dx} = \dfrac{1}{x+2}$ and $y(0)=1$, find an approximation of $y(1)$ using Euler's Method with two steps and step size $\Delta x = 0.5$.

4. Given the differential equation $\dfrac{dy}{dx} = x + y$ and $y(1)=3$, find an approximation of $y(2)$ using Euler's Method with two equal steps.

5. The curve passing through $(2,0)$ satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

6. Assume that f and f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2		

7. The table gives selected values for the derivative of a function *f* on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's Method with a step size of 0.5 is used to approximate $f(2)$, what is the resulting approximation?

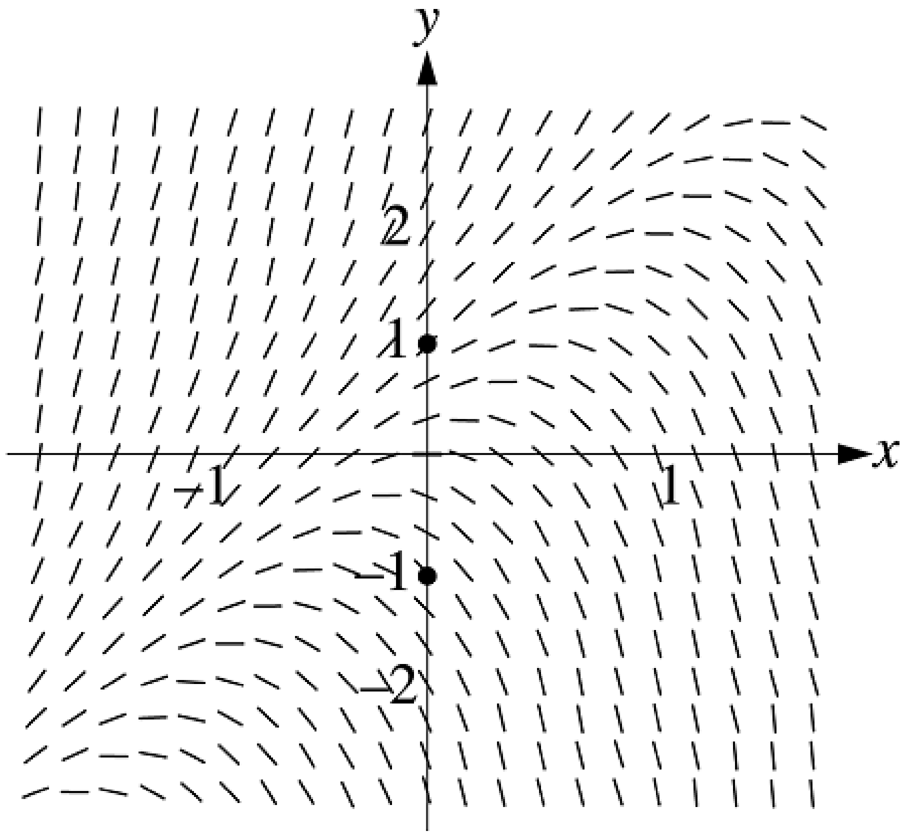
x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

8. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's Method, starting at $x = 0$ with two steps of equal size to approximate $f(-0.6)$.

9. AP 2002-5 (No Calculator)

Consider the differential equation: $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that passes through the point $(0,-1)$.



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0)=1$. Use Euler's method, starting at $x=0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

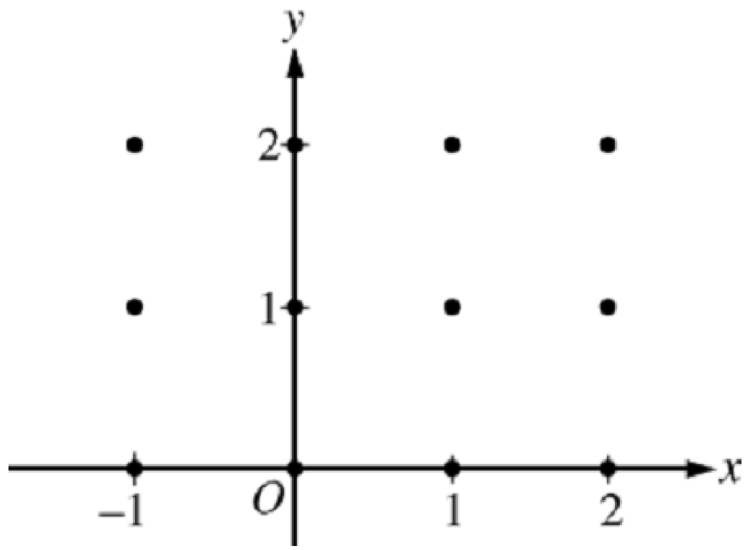
(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0)=0$. Does the graph of g have a local extremum at the point $(0,0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

10. AP 2005-4 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated and sketch the solution curve that passes through the point $(0,1)$.



(b) The solution curve that passes through the point $(0,1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the *y*-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of *x* and *y*. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.