① $\int x^2\cos x\,dx = h(x) - \int 2x\sin x\,dx$

u	dv	+/−
x^2	$\cos x$	+
$2x$	$\sin x$	−
2	$-\cos x$	+
0	$-\sin x$	−

u	dv	+/−
$2x$	$\sin x$	+
2	$-\cos x$	−
0	$-\sin x$	+

$x^2\sin x + 2x\cos x - 2\sin x = h(x) - [-2x\cos x + 2\sin x] + C$

$h(x) = x^2\sin x + \cancel{2x\cos x} - \cancel{2\sin x} - \cancel{2x\cos x} + \cancel{2\sin x} + C$

$\boxed{h(x) = x^2\sin x + C}$ $\boxed{B}$

② $\int x\sin(5x)\,dx$

u	dv	+/−
x	$\sin(5x)$	+
1	$-\frac{1}{5}\cos(5x)$	−
0	$-\frac{1}{25}\sin(5x)$	+

$= \boxed{-\frac{1}{5}x\cos(5x) + \frac{1}{25}\sin(5x) + C}$ $\boxed{B}$

③ $\int x\csc^2 x\,dx$

u	dv	+/−
x	$\csc^2 x$	+
1	$-\cot x$	−
0	$-\ln\lvert\sin x\rvert$	+

$= \boxed{-x\cot x + \ln\lvert\sin x\rvert + C}$

$\boxed{C}$

④ $\frac{dy}{dx} = 4x\ln x$

$\int dy = \int 4x\ln x\,dx$

u	dv	+/−
$\ln x$	$4x$	+
$\frac{1}{x}$	$2x^2$	−
$\frac{-1}{x^2}$	$\frac{2}{3}x^3$	+

tie off here

$y = 2x^2\ln x - \int\left(\frac{1}{x}\right)(2x^2)\,dx$

$\boxed{y = 2x^2\ln x - x^2 + C}$

choice $\boxed{C}$ fits this pattern

⑤ (a) $\int xe^{-x}\,dx$

u	dv	+/−
x	e^{-x}	+
1	$-e^{-x}$	−
0	e^{-x}	+

$= \boxed{-xe^{-x} - e^{-x} + C}$

⑤(b) $\int x^2\sin(\pi x)\,dx$

u	dv	+/−
x^2	$\sin\pi x$	+
$2x$	$-\frac{1}{\pi}\cos\pi x$	−
2	$-\frac{1}{\pi^2}\sin\pi x$	+
0	$\frac{1}{\pi^3}\cos\pi x$	−

$= \boxed{-\frac{x^2}{\pi}\cos(\pi x) + \frac{2x}{\pi^2}\sin(\pi x) + \frac{2}{\pi^3}\cos(\pi x) + C}$

(c) $\int \sin^{-1}x\,dx$

u	dv	+/−
$\sin^{-1}x$	1	+
$\frac{1}{\sqrt{1-x^2}}$	x	−

tie off here

$= x\sin^{-1}x - \int x(1-x^2)^{-1/2}\,dx$

$= x\sin^{-1}x - \frac{1}{2}(2)(1-x^2)^{1/2} + C$

corr pwr rule

$= \boxed{x\sin^{-1}x - \sqrt{1-x^2} + C}$

(d) $\int (\ln x)^2\,dx$

u	dv	+/−
$(\ln x)^2$	1	+
$\frac{2\ln x}{x}$	x	−

tie off

$= x\ln^2 x - \int\left(\frac{2\ln x}{x}\right)x\,dx$

$= x\ln^2 x - \int 2\ln x\,dx$

$= x\ln^2 x - 2[x\ln x - x] + C$

$= \boxed{x\ln^2 x - 2x\ln x + 2x + C}$

⑤ (e) $\int \arctan(4t)\,dt$

u	dv	+/-
$\arctan(4t)$	1	+
$\frac{4}{1+16t^2}$	t	−

tie off

$= t\arctan(4t) - \int \frac{4t}{1+16t^2}\,dt$

$= t\arctan(4t) - \frac{4}{32}\ln|1+16t^2| + C$

$= \boxed{t\arctan(4t) - \frac{1}{8}\ln|1+16t^2| + C}$

⑥ (a) $\int_0^{\pi} t\sin(3t)\,dt$

u	dv	+/-
t	$\sin(3t)$	+
1	$-\frac{1}{3}\cos(3t)$	−
0	$-\frac{1}{9}\sin(3t)$	+

$= -\frac{t}{3}\cos(3t) + \frac{1}{9}\sin(3t)\Big|_0^{\pi}$

$= \left(-\frac{\pi}{3}\underbrace{\cos(3\pi)}_{-1} + \frac{1}{9}\underbrace{\sin(3\pi)}_{0}\right) - (0+0)$

$= \boxed{\frac{\pi}{3}}$

⑥ (b) $\int_0^1 (x^2+1)(e^{-x})\,dx$

u	dv	+/-
x^2+1	e^{-x}	+
$2x$	$-e^{-x}$	−
2	e^{-x}	+
0	$-e^{-x}$	−

$= -(x^2+1)e^{-x} - 2xe^{-x} - 2e^{-x}\Big|_0^1$

$= e^{-x}(-x^2-1-2x-2)\Big|_0^1$

$= e^{-x}(-x^2-2x-3)\Big|_0^1$

$= e^{-1}(-1-2-3) - (e^0(-3))$

$= e^{-1}(-6)+3$

$= \boxed{3 - \frac{6}{e}}$

(c) $\int_1^e \frac{\ln x}{x^2}\,dx$

u	dv	+/-
$\ln x$	$\frac{1}{x^2}$	+
$\frac{1}{x}$	$-\frac{1}{x}$	−
$-\frac{1}{x^2}$		+

tie off

$= -\frac{\ln x}{x} + \int_1^e \frac{1}{x^2}\,dx$

$= -\frac{\ln x}{x} - \frac{1}{x}\Big|_1^e$

$= \left(-\frac{\ln e}{e} - \frac{1}{e}\right) - \left(-\frac{\ln 1}{1} - 1\right)$

$= \left(-\frac{2}{e}\right) + (1)$

$= \boxed{1 - \frac{2}{e}}$

(d) $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr = \int_0^1 (r^2)(r)(4+r^2)^{-1/2}\,dr$

u	dv	+/-
r^2	$r(4+r^2)^{-1/2}$	+
$2r$	$(\frac{1}{2})(2)(4+r^2)^{1/2}$	−
2		
0		

tie off

$= r^2\sqrt{4+r^2} - \int 2r(4+r^2)^{1/2}\,dr$

$= r^2\sqrt{4+r^2} - \frac{2}{3}(4+r^2)^{3/2}\Big|_0^1$

$= \sqrt{4+r^2}\left(r^2 - \frac{2}{3}(4+r^2)\right)\Big|_0^1$

$= \sqrt{4+r^2}\left(\frac{1}{3}r^2 - \frac{8}{3}\right)\Big|_0^1$

$= \left[\sqrt{5}\left(-\frac{7}{3}\right)\right] - \left[2\left(-\frac{8}{3}\right)\right]$

$= -\frac{7\sqrt{5}}{3} + \frac{16}{3}$

$= \boxed{\frac{16-7\sqrt{5}}{3}}$

(7) $\frac{dy}{dx} = x\sec^2 x$, $y = 1$ when $x = 0$

$\int dy = \int x\sec^2 x\,dx$

u	dv	+/–
x	$\sec^2 x$	+
1	$\tan x$	–
0	$-\ln\lvert\cos x\rvert$	+

$y = x\tan x + \ln\lvert\cos x\rvert + C$

at $(0,1)$: $1 = 0 + 0 + C$, $C = 1$

so $\boxed{y = x\tan x + \ln\lvert\cos x\rvert + 1}$

(8) $y = x\sin x$, $\pi \le x \le 2\pi$

* since $\sin x < 0$ $\forall x \in (\pi, 2\pi)$

$x\sin x < 0$ $\forall x \in (\pi, 2\pi)$, so

$\text{Area} = -\int_{\pi}^{2\pi} x\sin x\,dx$ or $\int_{2\pi}^{\pi} x\sin x\,dx$

u	dv	+/–
x	$\sin x$	+
1	$-\cos x$	–
0	$-\sin x$	+

$\text{Area} = -x\cos x + \sin x \Big|_{2\pi}^{\pi}$

$= (-\pi\cos\pi + \sin\pi) - (-2\pi\cos 2\pi + \sin 2\pi)$

$= \pi + 2\pi$

$= \boxed{3\pi}$

(9) $y = 2e^{-t}\cos t$, $t \ge 0$

Avg position on $t \in [0, 2\pi]$

$\text{Avg} = \frac{\int_0^{2\pi} 2e^{-t}\cos t\,dt}{2\pi - 0}$

$= \frac{2}{2\pi}\int e^{-t}\cos t\,dt$

u	dv	+/–
e^{-t}	$\cos t$	+
$-e^{-t}$	$\sin t$	–
e^{-t}	$-\cos t$	+

tie off

so $\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\,dt = \frac{1}{\pi}\left[e^{-t}\sin t - e^{-t}\cos t - \int_0 e^{-t}\cos t\,dt\right]$

$\frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\,dt = \frac{1}{\pi}e^{-t}\sin t - \frac{1}{\pi}e^{-t}\cos t - \frac{1}{\pi}\int e^{-t}\cos t\,dt$

(Like terms)

$\frac{2}{\pi}\int_0^{2\pi} e^{-t}\cos t\,dt = \frac{1}{\pi}\left(e^{-t}\sin t - e^{-t}\cos t\right)\Big|_0^{2\pi}$

$\text{Avg} = \frac{1}{\pi}\int_0^{2\pi} e^{-t}\cos t\,dt = \frac{1}{2\pi}\left[\left(e^{-2\pi}\sin 2\pi - e^{-2\pi}\cos 2\pi\right) - \left(e^0\sin 0 - e^0\cos 0\right)\right]$

$= \frac{1}{2\pi}\left[-e^{-2\pi} + e\right]$

$= \boxed{\frac{e - e^{-2\pi}}{2\pi}} \approx 0.4323$

(10) $\int \sin\sqrt{x}\,dx$, Let $u = x^{1/2}$, $x = u^2$, $dx = 2u\,du$

$= 2\int u\sin u\,du$

m	dv	+/–
u	$\sin u$	+
1	$-\cos u$	–
0	$-\sin u$	+

$= 2\left[-u\cos u + \sin u\right] + C$

$= \boxed{-2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C}$