Name____ Date__________________________ Period______

Worksheet 5.4—Integration by Parts
Show all work. No calculator unless stated.

Multiple Choice

1. If $\int x^2 \cos x\,dx = h(x) - \int 2x \sin x\,dx$, then $h(x) =$

(A) $2\sin x + 2x\cos x + C$ (B) $x^2 \sin x + C$ (C) $2x\cos x - x^2 \sin x + C$
(D) $4\cos x - 2x\sin x + C$ (E) $\left(2 - x^2\right)\cos x - 4\sin x + C$

2. $\int x \sin(5x)\,dx =$

(A) $-x\cos(5x) + \sin(5x) + C$ (B) $-\frac{x}{5}\cos(5x) + \frac{1}{25}\sin(5x) + C$ (C) $-\frac{x}{5}\cos(5x) + \frac{1}{5}\sin(5x) + C$
(D) $\frac{x}{5}\cos(5x) + \frac{1}{25}\sin(5x) + C$ (E) $5x\cos(5x) - \sin(5x) + C$

3. $\int x \csc^2 x\,dx =$

(A) $\frac{x\csc^3 x}{6} + C$ (B) $x\cot x - \ln|\sin x| + C$ (C) $-x\cot x + \ln|\sin x| + C$
(D) $-x\cot x - \ln|\sin x| + C$ (E) $-x\sec^2 x - \tan x + C$

4. The graph of $y = f(x)$ conforms to the slope field for the differential equation $\frac{dy}{dx} = 4x\ln x$, as shown. Which of the following could be $f(x)$?

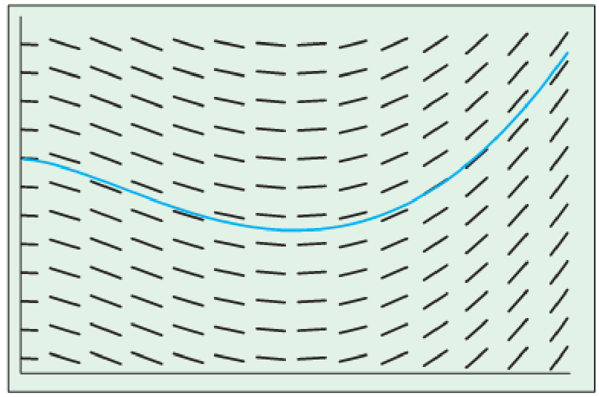
(A) $2x^2 \ln^2 x + 3$

(B) $x^3 \ln x + 3$

(C) $2x^2 \ln x - x^2 + 3$

(D) $(2x^2 + 3)\ln x - 1$

(E) $2x\ln^2 x - \frac{4\ln^3 x}{3} + 3$



[0, 2] by [0, 5]

Short Answer

5. Evaluate the following integrals.

(a) $\int xe^{-x}dx$

(b) $\int x^2 \sin(\pi x)dx$

(c) $\int \sin^{-1} x dx$

(d) $\int \ln^2 x dx$

(e) $\int \arctan 4t dt$

6. Evaluate the following definite integrals. Show the antiderivative. Verify on your calculator.

(a) $\int_0^{\pi} t\sin 3t\,dt$

(b) $\int_0^1 \left(x^2+1\right)e^{-x}\,dx$

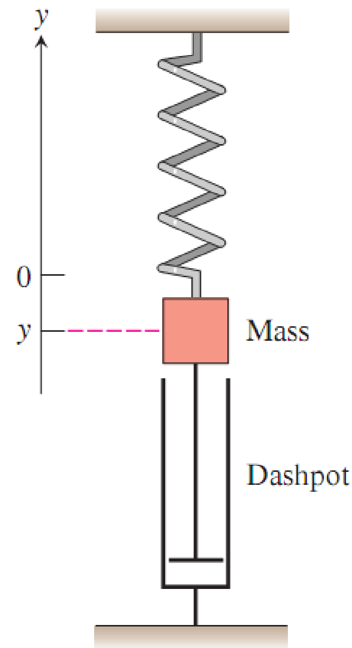
(c) $\int_1^e \frac{\ln x}{x^2}\,dx$

(d) $\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$ (Hint: let $r^3 = r^2 \cdot r$)

7. Solve: $\frac{dy}{dx} = x\sec^2 x$ and $y=1$ when $x=0$.

8. Find the area of the region enclosed by the *x*-axis and the curve $y = x\sin x$ for $\pi \le x \le 2\pi$.

9. A slowing force, symbolized by the “Dashpot” in the **figure at right**, slows the motion of the weighted spring so that the mass’s position at time t is given by $y = 2e^{-t}\cos t$, $t \geq 0$. Find the average position of the mass on the interval $t \in [0, 2\pi]$. Give an exact answer, then verify on your calculator.



10. Using u-substitution and then integration by parts, evaluate $\int \sin\sqrt{x}\,dx$.