Worksheet 5.4—Integration by Parts

Show all work. No calculator unless stated.

Multiple Choice

1. If \( \int x^2 \cos x \, dx = h(x) - \int 2x \sin x \, dx \), then \( h(x) = \)
   
   (A) \( 2\sin x + 2x \cos x + C \)  
   (B) \( x^2 \sin x + C \)  
   (C) \( 2x \cos x - x^2 \sin x + C \)  
   (D) \( 4\cos x - 2x \sin x + C \)  
   (E) \( (2 - x^2) \cos x - 4\sin x + C \)

2. \( \int x \sin (5x) \, dx = \)
   
   (A) \( -x \cos (5x) + \sin (5x) + C \)  
   (B) \( -\frac{x}{5} \cos (5x) + \frac{1}{25} \sin (5x) + C \)  
   (C) \( -\frac{x}{5} \cos (5x) + \frac{1}{5} \sin (5x) + C \)  
   (D) \( \frac{x}{5} \cos (5x) + \frac{1}{25} \sin (5x) + C \)  
   (E) \( 5x \cos (5x) - \sin (5x) + C \)

3. \( \int x \csc^2 x \, dx = \)
   
   (A) \( \frac{x \csc^3 x}{6} + C \)  
   (B) \( x \cot x - \ln |\sin x| + C \)  
   (C) \( -x \cot x + \ln |\sin x| + C \)  
   (D) \( -x \cot x - \ln |\sin x| + C \)  
   (E) \( -x \sec^2 x - \tan x + C \)
4. The graph of \( y = f(x) \) conforms to the slope field for the differential equation \( \frac{dy}{dx} = 4x \ln x \), as shown. Which of the following could be \( f(x) \)?

(A) \( 2x^2 \ln^2 x + 3 \)
(B) \( x^3 \ln x + 3 \)
(C) \( 2x^2 \ln x - x^2 + 3 \)
(D) \( (2x^2 + 3) \ln x - 1 \)
(E) \( 2x \ln^2 x - \frac{4\ln^3 x}{3} + 3 \)

Short Answer

5. Evaluate the following integrals.

(a) \( \int xe^{-x}dx \)  
(b) \( \int x^2 \sin(\pi x)dx \)  
(c) \( \int \sin^{-1}xdx \)

(d) \( \int \ln^2 xdx \)  
(e) \( \int \arctan 4tdt \)
6. Evaluate the following definite integrals. Show the antiderivative. Verify on your calculator.

(a) \[ \int_{0}^{\pi} t \sin 3t \, dt \]

(b) \[ \int_{0}^{1} \left( x^2 + 1 \right) e^{-x} \, dx \]

(c) \[ \int_{1}^{e} \frac{\ln x}{x^2} \, dx \]

(d) \[ \int_{0}^{1} \frac{r^3}{\sqrt{4 + r^2}} \, dr \] (Hint: let \( r^3 = r^2 \cdot r \))

7. Solve: \( \frac{dy}{dx} = x \sec^2 x \) and \( y = 1 \) when \( x = 0 \).

8. Find the area of the region enclosed by the \( x \)-axis and the curve \( y = x \sin x \) for \( \pi \leq x \leq 2\pi \).
9. A slowing force, symbolized by the “Dashpot” in the figure at right, slows the motion of the weighted spring so that the mass’s position at time $t$ is given by $y = 2e^{-t} \cos t$, $t \geq 0$. Find the average position of the mass on the interval $t \in [0, 2\pi]$. Give an exact answer, then verify on your calculator.

10. Using $u$-substitution and then integration by parts, evaluate $\int \sin \sqrt{x} \, dx$. 
