Name____ Date_______________________ Period_____

Worksheet 5.5—Partial Fractions & Logistic Growth
Show all work. No calculator unless stated.

Multiple Choice

1. The spread of a disease through a community can be modeled with the logistic equation $y = \dfrac{600}{1 + 59e^{-0.1t}}$, where y is the number of people infected after t days. How many people are infected when the disease is spreading the fastest?

(A) 10 (B) 59 (C) 60 (D) 300 (E) 600

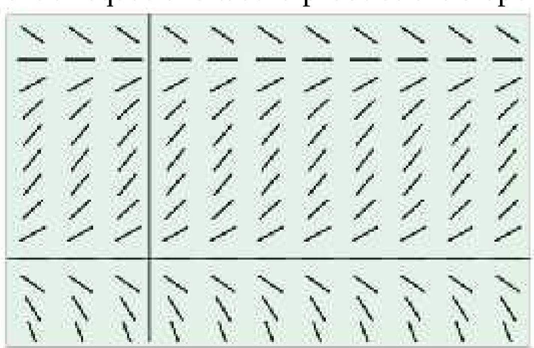
2. The spread of a disease through a community can be modeled with the logistic equation $y = \dfrac{0.9}{1 + 45e^{-0.15t}}$, where y is the proportion of people infected after t days. According to the model, what percentage of people in the community will not become infected?

(A) 2% (B) 10% (C) 15% (D) 45% (E) 90%

3. $\displaystyle\int_2^3 \frac{3}{(x-1)(x+2)}\,dx =$

(A) $-\dfrac{33}{20}$ (B) $-\dfrac{9}{20}$ (C) $\ln\left(\dfrac{5}{2}\right)$ (D) $\ln\left(\dfrac{8}{5}\right)$ (E) $\ln\left(\dfrac{2}{5}\right)$

4. Which of the following differential equations would produce the slope field shown below?



[–3, 8] by [–50, 150]

(A) $\dfrac{dy}{dx} = 0.01x(120-x)$ (B) $\dfrac{dy}{dx} = 0.01y(120-y)$ (C) $\dfrac{dy}{dx} = 0.01y(100-x)$

(D) $\dfrac{dy}{dx} = \dfrac{120}{1+60e^{-1.2x}}$ (E) $\dfrac{dy}{dx} = \dfrac{120}{1+60e^{-1.2y}}$

5. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population is $P(0) = 3000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2500 (B) 3000 (C) 4200 (D) 5000 (E) 10,000

6. Suppose a population of wolves grows according to the logistic differential equation $\frac{dP}{dt} = 3P - 0.01P^2$, where P is the number of wolves at time t, in years. Which of the following statements are true?

I. $\lim_{t\to\infty} P(t) = 300$

II. The growth rate of the wolf population is greatest when $P = 150$.

III. If $P > 300$, the population of wolves is increasing.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

7. $\int \frac{7x}{(2x-3)(x+2)}\,dx =$

(A) $\frac{3}{2}\ln|2x-3|+2\ln|x+2|+C$ (B) $3\ln|2x-3|+2\ln|x+2|+C$ (C) $3\ln|2x-3|-2\ln|x+2|+C$

(D) $-\frac{6}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$ (E) $-\frac{3}{(2x-3)^2}-\frac{2}{(x+2)^2}+C$

8. $\int \frac{2x}{x^2+3x+2}dx =$

(A) $\ln|x+2|+\ln|x+1|+C$ (B) $\ln|x+2|+\ln|x+1|-3x+C$ (C) $-4\ln|x+2|+2\ln|x+1|+C$

(D) $4\ln|x+2|-2\ln|x+1|+C$ (E) $2\ln|x|+\frac{2}{3}x+\frac{1}{2}x^2+C$

Short Answer/Free Response

Work the following on notebook paper.

9. Suppose the population of bears in a national park grows according to the logistic differential equation $\frac{dP}{dt} = 5P - 0.002P^2$, where P is the number of bears at time t in years.

   (a) If $P(0) = 100$, then $\lim_{t \to \infty} P(t) =$ _____________. Sketch the graph of $P(t)$. For what values of P is the graph of P increasing? decreasing? Justify your answer.

   (b) If $P(0) = 1500$, $\lim_{t \to \infty} P(t) =$ _____________. Sketch the graph of $P(t)$. For what values of P is the graph of P increasing? decreasing? Justify your answer.

   (c) If $P(0) = 3000$, $\lim_{t \to \infty} P(t) =$ _____________. Sketch the graph of $P(t)$. For what values of P is the graph of P increasing? decreasing? Justify your answer.

   (d) How many bears are in the park when the population of bears is growing the fastest? Justify your answer.

10. (Calculator Permitted) A population of animals is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = 0.01P(100 - P)$, where t is measured in years.

(a) If $P(0) = 20$, solve for P as a function of t.

(b) Use your answer to (a) to find P when $t = 3$ years. Give exact and 3-decimal approximation.

(c) Use your answer to (a) to find t when $P = 80$ animals. Give exact and 3-decimal approximation.

11. (Calculator Permitted) The rate at which a rumor spreads through a high school of 2000 students can be modeled by the differential equation $\frac{dP}{dt} = 0.003P(2000 - P)$, where P is the number of students who have heard the rumor t hours after 9AM.

(a) How many students have heard the rumor when it is spreading the fastest?

(b) If $P(0) = 5$, solve for P as a function of t.

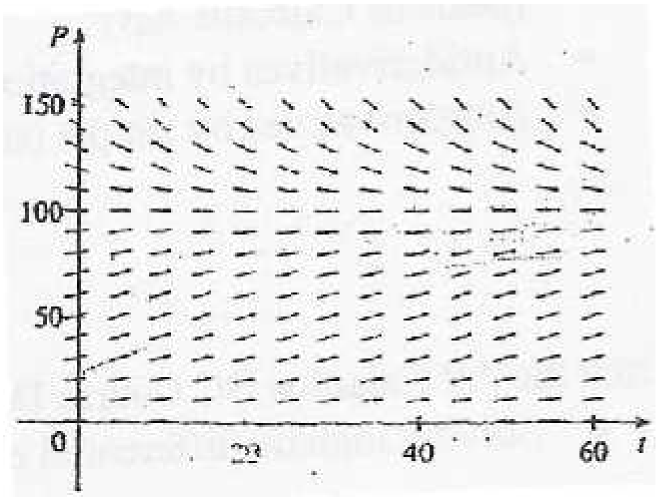
(c) Use your answer to (b) to determine how many hours have passed when the rumor is spreading the fastest. Give exact and 3-decimal approximation.

(d) Use your answer to (b) to determine the number of people who have heard the rumor after two hours. Give exact and 3-decimal approximation.

12. Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$ where t is measured in weeks.

(a) What is the carrying capacity/limit to growth?

(b) A slope field for this equation is shown below.



I. Where are the slopes close to zero?

II. Where are they largest?

III. Which solutions are increasing?

IV. Which solutions are decreasing?

(c) Use the slope field to sketch solutions for initial populations of 20, 60, and 120.

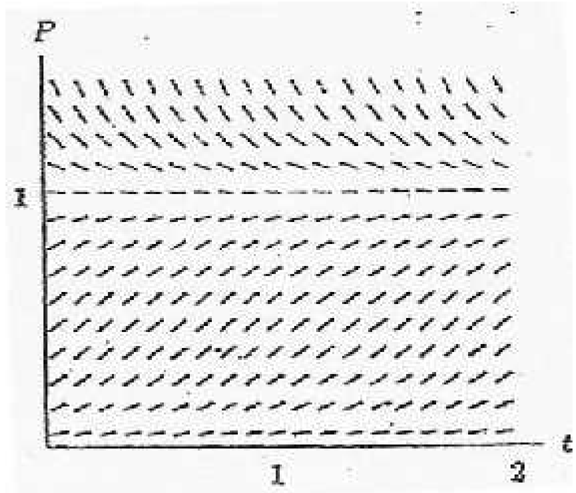
I. What do these solutions have in common?

II. How do they differ?

III. Which solutions have inflection points?

IV. At what population level do these inflection points occur?

13. The slope field show below gives general solutions for the differential equation given by $\frac{dP}{dt} = 3P - 3P^2$.



(a) On the graph above, sketch three solution curves showing three different types of behavior for the population *P*.

(b) Describe the meaning of the shape of the solution curves for the population.

I. Where is P increasing?

II. Where is P decreasing?

III. What happens in the long run (for large values of *t*)?

IV. Are there any inflection points? If so, where?

V. What do the inflection points mean for the population?

14. (Calculator Permitted) Newton's Law of Cooling: Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. A detective finds a murder victim at 9 A.M. The temperature of the body is measured at $90.3^\circ F$. One hour later, the temperature of the body is $89.0^\circ F$. The temperature of the room has been maintained at a constant $68.0^\circ F$

(a) Assuming the temperature, *T*, of the body obeys Newton's Law of Cooling, write a differential equation for *T*, in degrees Fahrenheit, as a function of *t* hours.

(b) Solve the differential equation to estimate the time the murder occurred.

(c) Call the cops and let them know.