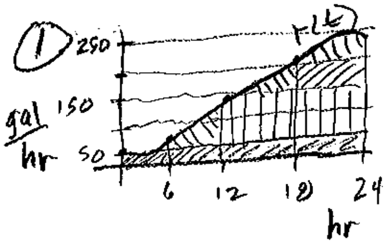
WS 8.1 – Integral as Net Δ KEY CALCULUS MAXIMUS

(1)



Total gallons $t \in [0,24]$

$= \int_0^{24} r(t)\,dt$

$\approx 24(50) + 6(50) + 12(100) + 6(50) + 6(50) + \text{little more}$

$= 3300 + \text{little more}$

3600 is closest to this value so the answer is $\boxed{C}$ $\approx \boxed{3600 \text{ gallons}}$

(2) $v(0) = 5$, $v(15) = ?$

$v(15) = v(0) + \int_0^{15} a(t)\,dt$

$\approx 5 + \left[\underbrace{\tfrac{1}{2}(3)\big(4 + 2(8) + 2(6) + 2(9) + 2(10) + 10\big)}_{\text{shows trapezoidal method}}\right]$

$= 5 + 120$

$= \boxed{125 \text{ ft/sec}}$ $\boxed{D}$

(3) Customers arriving for $t \in [0,60]$

$= \int_0^{60} F(t)\,dt = 724.645 \approx \boxed{725 \text{ customers}}$

$\boxed{B}$

(4) Pollution removed from '95 to '05, $t \in [0,10]$

$= \int_0^{10} 20e^{-0.5t}\,dt = 39.730 \approx \boxed{40 \text{ tons}}$

$\boxed{A}$

(5) Oil consumed for $t \in [0,10]$

$= \int_0^{10} r(t)\,dt = 638.905$ millions of barrels

$\boxed{D}$

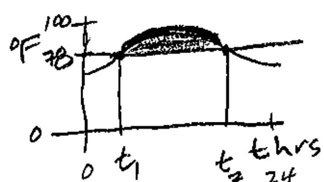
Bonus info!!

(6) $F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right)$, $0 \le t \le 24$ hrs

*(put F(t) in Y1 in calculator)

(a) $\text{AvgTemp} = \dfrac{\int_6^{14} F(t)\,dt}{14 - 6} = \boxed{87.161°F}$

(b) $F(t) = 78$ *solve graphically finding pts. of intersection on window $t \in [0,24]$



From calculator

$t_1 = 5.230$ hrs $= A$ *store as A

$t_2 = 18.769$ hrs $= B$ *store as B

So A/C ran to cool the house for all $t \in [5.230, 18.769]$ hrs

concluding statement (with units!)

(c) Total Cost to Cool

$= \dfrac{\$0.05}{\cancel{hr} \cdot deg} \cdot \int_A^B (F(t) - 78) \cdot dt$ hr·deg aggregate units

°F · hr

$= 5.096 \approx \boxed{\$5.09 \text{ or } \$5.10}$ to cool house

(7) $E(t) = y1$, $L(t) = y2$ (in calculator)

(a) People entered $= \int_9^{17} E(t)\,dt = 6004.270$

$\approx \boxed{6004 \text{ people}}$

(b) Revenue

$= 15 \cdot \int_9^{17} E(t)\,dt + 11 \cdot \int_{17}^{23} E(t)\,dt$

$= 104048.1652$

$\approx \boxed{\$104{,}048 \text{ collected}}$

(c) * $H(17) = \int_9^{17} (E(x) - L(x))\,dx$ which tells us that at 5:00pm, 3725 people were in the park.

* $H'(t) = E(t) - L(t)$, $H'(17) = E(17) - L(17)$

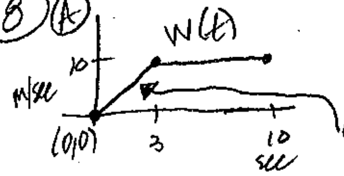
$H'(17) = -380.281$ people per hour.

which means at 5:00PM, people are leaving the park at a rate of 380.281 people per hour.

(d) Maximize $H(t)$: $H'(t) = 0$ when $E(t) = L(t)$, which is at $t = 15.794$ hrs $= A$ *store as A

*CLOSED INTERVAL TEST: $t \in [9,23]$ $H(9) = 0$, $H(23) = 1.013$, $H(A) = 3950.68$, $\boxed{\text{So Park attendance is a Max at } t = 15.794 \text{ hrs after 9am.}}$

(8) (A)



eq: $y = 0 + \frac{10}{3}(x-0) = \frac{10}{3}x$

(B) $v(t) = \frac{24t}{2t+3}$

(a) Runner A: $w(2) = \frac{10}{3}(2) = \boxed{\frac{20}{3} \text{ m/sec}} \approx 6.666$ m/sec

Runner B: $v(2) = \frac{24(2)}{7} = \boxed{\frac{48}{7} \text{ m/sec}} \approx 6.857$ m/sec

(b) Runner A: $w'(2) = \boxed{\frac{10}{3} \text{ m/sec}^2}$

Runner B: $v'(2) = \boxed{1.469 \text{ m/sec}^2}$ ← from calculator

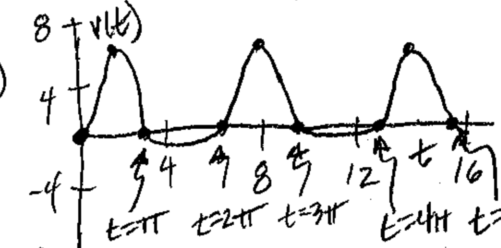
(c) Total Distance (no cancellation)

Runner A: Dist $= \int_0^{10} w(t)\,dt$
$= \frac{1}{2}(10+7)(10)$ ← one, big trapezoid
$= \boxed{85 \text{ meters}}$

Runner B: Dist $= \int_0^{10} v(t)\,dt$
$= \boxed{83.336 \text{ meters}}$ ← from calculator

(9) $v(t) = e^{2\sin t} - 1$, $t \in [0,16]$, when $t=0, x=0$ (initial condition)

(a)



*t-intercepts at multiples of π.

(b) Particle moves left when $v(t)<0$. This occurs on the intervals $\boxed{t \in (\pi, 2\pi) \cup (3\pi, 4\pi) \cup (5\pi, 16]}$

(c) Total Distance $= \int_0^{\pi} v(t)\,dt - \int_{\pi}^{4} v(t)\,dt$ OR (since this a calculator permitted question)

or

Total Distance $= \int_0^4 |v(t)|\,dt = \boxed{10.542}$

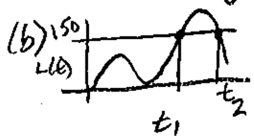
(d) Since particle starts at origin ($x=0$), it will only return to the origin if its displacement equals zero for any interval in $[0,16]$.

* Since $v(t)>0$ on $t\in(0,\pi)$ and $v(t)<0$ on $t\in(\pi,2\pi)$ we compute $\int_0^{2\pi} v(t)\,dt$ to be $8.039 > 0$. Because of symmetry of the graph, $\int_0^t v(x)\,dx \neq 0$ for any $t \in (0,16]$, so the particle does not return to the origin on $t\in(0,16]$.

(10) $L(t) = 60\sqrt{t}\,\sin^2\left(\frac{t}{3}\right)$, $t\in[0,18]$

(a) Cars turning left $= \int_0^{18} L(t)\,dt = 1657.823 \approx \boxed{1658 \text{ cars}}$

(b)



$L(t)=150$ for $t_1 = 12.428 = A$ (*store as A) and $t_2 = 16.121 = B$ (*store as B)

So $L(t) \geq 150$ for $t \in [12.428, 16.121]$ hours.

*Avg L-value $= \frac{\int_A^B L(t)\,dt}{B-A} = \boxed{199.426 \text{ cars per hour}}$

(c) $L(t)$ appears to be a max near $t=14$ hrs, so cars turning left in 2-hr interval will be near max on $t \in [14-1, 14+1] = [13,15]$ hrs.

Test: Index $= (500)\int_{13}^{15} L(t)\,dt = 215{,}965.7002$ cars which exceeds 200,000 cars, so YES! A traffic signal is required!

(11)

t (hrs)	0	1	3	4	7	8	9
$L(t)$ (ppl)	120	156	176	126	150	80	0

12:00 pm = NOON → $t=0$
$t \in [0,9]$ hrs

(a) $L'(5.5) \approx \underbrace{\dfrac{150-126}{7-4}}_{\text{difference quotient!}} = 8 \underbrace{\text{ppl/hr}}_{\text{unit!}}$

squiggles

(b) Avg people $= \dfrac{\int_0^4 L(t)\,dt}{4-0} \approx \left(\frac{1}{4}\right)\left[\overbrace{\left(\frac{1}{2}\right)\left(1(120+156)+2(156+176)+1(176+126)\right)}^{\text{conveys trapezoidal method}}\right]$ people

$= \boxed{155.25 \text{ people}}$

(c) since $L(t)$ is twice-differentiable, $L(t)$ and $L'(t)$ are continuous and the MVT applies to $L(t)$ and the IVT apelies to both $L(t)$ and $L'(t)$.

* Avg rate of change of $L(t)$ on $[1,3]$ $= \dfrac{176-156}{3-1} > 0$
* Avg rate of change of $L(t)$ on $[3,4]$ $= \dfrac{126-176}{4-3} < 0$
* So by the MVT, $L'(t)$ must be positive for some $t \in (1,3)$ and $L'(t)$ must be negative for some $t \in (3,4)$.
* Since $L'(t)$ is continuous on $[1,4]$, by the IVT there must be a $t \in [1,4]$ such that $L'(t) = 0$.
* Similar arguments can be made about $L'(t)=0$ on $t \in [3,7]$ and $t \in [4,8]$ so $L'(t)=0$ for at least 3 values of $t \in [0,9]$.

(d) $r(t) = 550te^{-t/2}$

Tickets sold $= \int_0^3 r(t)\,dt = 972.784 \approx \boxed{973 \text{ tickets}}$ sold by 3 P.M.

(12)

t	0	2	5	9	10	min
$H(t)$	66	60	52	44	43	°C

(a) $H'(3.5) \approx \underbrace{\frac{52-60}{5-2}}_{\text{difference quotient}} = \boxed{-\frac{8}{3}\ °C/min}$ ✓

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ represents the average temperature of the tea, ✓ in degrees Celsius, over the 10-minute interval from $t=0$ min to $t=10$ min.

$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \left(\frac{1}{10}\right)\left(\frac{1}{2}\right)\left[2(66+60)+3(60+52)+4(52+44)+1(44+43)\right]$ °C ✓ method

$= \boxed{52.95°C}$ ✓

(c) $\int_0^{10} H'(t)\,dt = H(t)\Big|_0^{10} = H(10) - H(0) = 43 - 66 = \boxed{-23°C}$ ✓

This means the tea's temperature drops 23°C during the 10-minute interval from $t=0$ min to $t=10$ min. ✓

(d) $B(10) = B(0) + \int_0^{10} B'(t)\,dt$ ✓

$= 100 + \int_0^{10} \left(-13.84e^{-0.173t}\right)dt = \underline{34.182°C} = A$ (*Store as A)

✓ ← Biscuits' temp at $t=10$min

$H(10) = 43°C$ ← tea's temp at $t=10$ min

* the biscuits, after 10 minutes, are $43°C - 34.182°C = \boxed{8.817247202°C}$ ✓ cooler than the tea.